Social learning from actions and outcomes

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Abstract

We study a two-player game of strategic timing of irreversible investments where returns depend on an uncertain state of the world. Agents learn about the state both through privately observed signals and from each other's actions and experience. We show that, if there is sufficient initial optimism about the state and signals are none too informative, private information may increase ex ante welfare by mitigating agents' incentives for free-riding.

1 Introduction

Is transparency good for innovation? The question is at times controversially debated in the public square, often seeming to pit industry executives against regulators.[1] We propose a simple model of innovation in which, say, two pharmaceutical companies decide on making an irreversible investment, such as putting a certain medical substance on the market. The quality of the investment option is the same for both companies but initially unknown. Either company has some initial private information about its quality. As long as the substance is marketed, it generates some positive flow payoff. But, if the substance is of low quality, this will be discovered during its marketing phase after an exponentially distributed time, e.g, because a consumer is harmed. In this case, the drug is banned, the game ends, and the company responsible for causing the harm is fined so heavily that it regrets having marketed the drug.

We show that, in our setting, keeping the companies' initial private information private may be beneficial for welfare.[2] Indeed, on account of the irreversibility of investments, agents wait too long before they invest in the hope that the other one provides free information. Excessive delay *before investments* is the result of *leadership-aversion*. Indeed, the follower can get additional information concerning the quality of the investment by observing the leader's experience; moreover,

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[1]See e.g. the report *Ending Drug Trial Secrecy Is a Boon For R & D, Says EU Agency* (Reuters, 2013).

[2]We do not include consumers in our model. Thus, if we wanted to include them in our welfare calculus, we should have to assume that the fine is set in such a way as to make the firm fully internalize the harm caused.

conditionally on the project being bad, the leader is more likely to be subjected to the fine. *After the first agent has invested,* the other agent trades off the forgone returns from investing with the benefit of waiting for more information, without taking into account the positive externality her investment would exert on her partner, leading to a delay in the follower's investment through a classical *free-riding effect*.

As our main result shows, not knowing one's partner's type may mitigate both sources of inefficiency. Indeed, when agents have private information, delaying their investments has drawbacks. An agent can only benefit from waiting to invest if the other agent invests first. However, one agent's delay may signal bad information to the other, who, in response, may be less willing to invest, thereby reducing the first agent's incentive to delay. As a result, an agent who received a bad signal may have an incentive to invest without delay nonetheless, in order to encourage the other agent to invest earlier. A second drawback results from each agent's uncertainty about the other agent's behavior. For example, an agent who received a good signal may wait in the hope that the other agent invests first. The other agent, however, may have received a bad signal and therefore never invest. Or an agent may delay her investment despite having received a good signal, so as to learn whether or not the other agent invests. However, by delaying her investment, she discourages future investment by the other agent, even if the latter has received a good signal. These negative effects of delay under private information can induce agents to invest earlier, and thereby reduce free-riding and leadership aversion. This reduction increases ex-ante social welfare as long as it does not result in inefficient over-investment. For example, an agent with a good signal may invest not knowing the other agent's signal, but regret investing if she learns that the other agent's signal is bad. In cases like these, it might be socially desirable to make agents' private information public.

Several papers have investigated the role of private information in games of informational externalities.[3] Rosenberg et al. (2013) investigate a game of strategic experimentation with exponential two-armed bandits, where players' action choices are public information, while the outcomes of these actions are private information. A player's switch to the safe arm is irreversible. They show that, in their setting, public information is unequivocally good for welfare. Their setup differs from ours inter alia by the fact that their players accrue private information over time, while ours are privately informed at the outset. Furthermore, there are no payoff externalities in Rosenberg et al. (2013), so that, once they have irreversibly exited the game, their players do not care about their partner's behavior any longer. Our players, by contrast, decide on when irreversibly to *enter* the market, and thus might have incentives to bias their partner's entry decision even conditional on their entering themselves.

Heidhues et al. (2015) investigate the problem with public reversible actions and private payoffs,

[3]The problem of strategic information acquisition in bandit games has been introduced by Bolton and Harris (1999) in a Brownian-motion environment. Keller et al. (2005) have extended the analysis to a Poisson setting, while Keller and Rady (2015) have introduced "bad news" Poisson events. Private information in this setting has also been analyzed in Rosenberg et al. (2007).

while allowing for cheap-talk communication among players. They show that equilibria with public information can always be replicated under private information, so that private information is unequivocally good for welfare. This conclusion heavily depends on their assumption that players can communicate among each other. In contrast, Bonatti and Hörner (2011), analyze the case of unobservable and reversible actions and observable outcomes, and find that private information boosts welfare in their setting. The reason is that, with observable actions, a deviation by a player may render the other players more optimistic, and hence more willing to pick up the slack caused by the deviation. This in turn makes deviating more attractive under public information.

In Chamley and Gale (1994) and Murto and Välimäki (2011, 2013), information is dispersed throughout society, and agents decide when to take an irreversible action. As in our setting, information is inefficiently aggregated because investors have an incentive to delay their decision so as to acquire more information by observing the behavior of others. In contrast to our setting, however, the players do not observe the results of their partners' experimentation directly. Moreover, once an agent takes an action, he effectively exits the game, and is no longer affected by others' decisions. In our setting, by contrast, players continue to be affected by others' actions even after taking their irreversible action, and thus have an incentive to influence their partner's actions and beliefs when making their irreversible investment decision.

A closely related paper is Décamps and Mariotti (2004), who also study a two-player game of irreversible investments. Private information in their setting, however, pertains to the players' idiosyncratic investment costs, while all information concerning the common quality of the investment opportunity is public. Since, as in our setting, players get additional (public) information after the other player has invested, they prefer the role of follower and thus have an incentive to convince each other that their own costs of investment are high. Once they have made their irreversible investment decision, Décamps and Mariotti (2004)'s players do not care about their partner's actions any longer, as externalities are purely informational in their setting. In our setting, by contrast, there is a payoff externality, as only one player will incur the costs of an accident. Thus, even conditionally on having made the irreversible investment decision, a player prefers her partner to invest as soon as possible in our setting. Thus, our players have incentives to render their partners as optimistic as possible concerning the common investment prospects.

Moscarini and Squintani (2010) consider a model of a winner-takes-all R&D competition in which firms observe an initial private signal about the unknown type of a research project, which is drawn from a continuous distribution. Over time, firms learn about the project's type from their competitor's actions and the lack of success in the past, deciding when to exit irreversibly. They show that the aggregate duration of experimentation is longer under private information, when firms may also exit simultaneously, a non-generic outcome under public information.

The potential welfare improvement of private information is related to the "smoothing effect of uncertainty" (Morris and Shin, 2002). Teoh (1997) demonstrates this effect in a model of public-good provision, where the marginal return to agents' investments is determined by an uncertain state of the world. The author shows that non-disclosure of information may increase ex-ante welfare when the investment has marginally diminishing returns, because the loss resulting from a reduction in investment after the release of bad news outweighs the benefits from increased investment when the information is favorable. This is the same mechanism that drives the main result in our paper: when bad news is publicly disclosed, free-riding and leadership-aversion increase, leading to an over-proportional reduction in the expected value of investment.

2 Model

There are two agents, indexed $i = 1, 2$. Time $t \in \mathbb{R}_+$ is continuous, with an infinite horizon, and future payoffs are discounted at the common discount rate r. Each agent decides when to invest in a project which generates a payoff stream that depends on an unknown state of the world $\theta \in \{G, B\}$, which is either good ($\theta = G$) or bad ($\theta = B$). The investment is irreversible. In either state, the project yields a certain flow return $y > 0$. In state B, an accident occurs at a random time corresponding to the first jumping time of a Poisson process with parameter $\gamma > 0$. An accident never occurs in state H. The game ends at the time of the first accident. The agent suffering the first accident incurs a lump-sum cost of $c > 0$.

At the outset, each agent $i = 1, 2$ observes a private binary signal $s^i \in \{g, b\}$, which provides information about the realization of the state. The agents share a private initial belief that the state is good, which we denote by $p_0 \in (0, 1)$. Either agent's signal is correct (i.e., equals g [b] in state G [B]) with probability $\rho \in (1/2, 1)$. Conditionally on θ, the signal realizations are independent between the agents.

We model the continuous-time environment as a stopping game in two phases. At the beginning of the first phase, the agents play an attrition game in which either decides how long to wait before making the investment, conditionally on the event that the other agent has not yet invested. The initial stage ends after the first agent (the *leader*) invests. In the second phase, the other agent (the *follower*) then decides how long to wait before investing as well.

Formally, a non-terminal history h at a time t when neither agent has yet made an investment is denoted by $h = t$. A non-terminal history at a time t such that agent $i \in \{1, 2\}$ has invested at some time $\tau \in [0, t)$, is denoted by $h = (t, (i, \tau))$. Denote by $p(s_i, h)$ an agent's posterior belief about the state at history h, given she has observed the signal realization s_i. Note that exogenous randomness arrives only in the form of accidents, but, since these immediately end the game, the belief $p(s_i, h)$ is deterministic, given the initial belief p_0.

Either agent's strategy in the first phase is characterized by a cumulative distribution function $F_{i,0}(s_i, t)$ representing the probability that agent i with signal s_i invests before time t conditional on the other agent $-i$ not having invested up to time t. For the second phase, each agent's strategy is characterized by a family of c.d.f.'s $\{F_{i,1}(s_i, t, \tau)\}_{t \geq 0}$ where $F_{i,1}(s_i, t, \tau)$ is the probability that agent i, in the role of the follower, stops at time $t \geq \tau$, conditional on (i) the other agent having invested at time τ, and (ii) no accident having occurred up to time t. Our solution concept is symmetric perfect Bayesian equilibrium.

We denote by $q_0(s) := p(s,0)\rho + (1-p(s,0))(1-\rho)$ for each $s = g, b$ the prior probability an agent with signal s assigns to the event that the other agent has observed a good signal. We shall furthermore write $p_0(s) := p(s,0)$, and $p_0(s^1, s^2)$ for the updated belief at time $t = 0$, given the initial belief p_0 and signal realizations (s^1, s^2).

3 Equilibrium analysis

3.1 Public information

We first consider the case in which both signals are publicly observable. The agents then share a common initial belief about the state, which we denote by $\check{p}_0 := p_0(s^1, s^2)$. Once both agents have made the investment, there are no additional moves to make, and the game continues indefinitely or until an accident occurs.

Each agent benefits from the other agent's investment, because she may learn the true state from the other agent's accident without incurring a loss herself. The expected net present value for the leader at belief p is

$$v_l(p,\tau) = py + (1-p)\left((1-e^{-(r+\gamma)\tau})\lambda_1 + e^{-(r+\gamma)\tau}\lambda_2\right)(y-\gamma c), \tag{1}$$

when the other agent delays her investment by τ, where $\lambda_k := r/(r+k\gamma)$ for each $k = 1, 2$. Similarly, the expected present value for the follower is

$$v_f(p,\tau) = e^{-r\tau}py + e^{-(r+\gamma)\tau}(1-p)\lambda_2(y-c\gamma). \tag{2}$$

To state the following lemma, which reports basic properties of the functions v_l and v_f, we define the log-likelihood ratio $\phi(p) := \ln\left(\frac{p}{1-p}\right)$.

Lemma 1. *The function $v_l(p,\tau)$ is linearly increasing in p, convex and decreasing in τ for every $p \in (0,1)$ and supermodular in (p,τ). The function $v_f(p,\tau)$ is linearly increasing in p and has a single peak in τ at*

$$\tau^*(p) = \begin{cases} \left(\phi(p_f^*) - \phi(p)\right)/\gamma & \text{if } p < p_f^* \\ 0 & \text{if } p \geq p_f^* \end{cases} \tag{3}$$

for every $p \in (0,1)$*, where*

$$p_f^* = \frac{\lambda_2(\gamma c - y)}{\lambda_1 y + \lambda_2(\gamma c - y)}. \tag{4}$$

All proofs are found in the appendix. We write $v_f^*(p) = v_f(p, \tau^*(p))$ and $v_l^*(p) = v_l(p, \tau^*(p))$ for the values of the leader and the follower, respectively, given the follower uses the optimal delay. Since τ^* is weakly decreasing in p, and v_l and v_f are strictly increasing in p as well as decreasing in τ, it follows that v_l^* and v_f^* are strictly increasing functions in p. Moreover, v_l^* is continuous, positive if $p = 1$, and negative if $p = 0$. Hence, it has a unique root on $(0,1)$, which we denote by p_l^*. Further, denote by $\underline{p}$ the threshold above which the value of both agents investing immediately is positive:

$$\underline{p} = \frac{\lambda_2(\gamma c - y)}{y + \lambda_2(\gamma c - y)}.$$

Lemma 2. $0 < \underline{p} < p_l^* < p_f^* < 1.$

It follows from Lemma 1, that, in any equilibrium, both agents invest immediately if $\check{p}_0 \geq p_f^*$. Indeed, if the belief exceeds this threshold, then it is optimal for the follower not to delay investing, so that it is optimal for each agent to invest immediately. If $\check{p}_0 \leq p_l^*$, neither agent invests in any equilibrium, by definition of p_l^*. If $p_l^* < \check{p}_0 < p_f^*$, there can be no symmetric equilibrium in pure strategies, as the best response to the other agent's investment would be not to invest and vice-versa. The same argument rules out atoms in mixed-strategy equilibrium. Thus, a symmetric equilibrium must be in atomless mixed strategies, with each agent investing at a rate that renders the other agent indifferent between investing immediately and delaying her investment by any length of time. As long as neither agent has made the investment, no new information becomes available, so that the agents' equilibrium flow rate of investment β is constant over time. We can immediately calculate the equilibrium investment rate, using the fact that each agent must be indifferent between making the investment immediately and never making the investment, i.e.,

$$v_l^*(\check{p}_0) = \int_0^\infty \beta e^{-(\beta+r)t} v_f^*(\check{p}_0) dt.$$

Solving the equation for β gives the equilibrium investment rate as a function of the common belief $\check{p}_0$:

$$\beta^*(\check{p}_0) = \max\left\{\frac{r v_l^*(\check{p}_0)}{v_f^*(\check{p}_0) - v_l^*(\check{p}_0)}, 0\right\}. \tag{5}$$

Observe that the rate of investment is positive whenever the value of becoming the leader is greater than 0, because it follows from $\check{p}_0 < p_f^*$ and Lemma 1 that the denominator of β^* is always strictly positive. Moreover, the difference between the value of the follower and the value of the leader converges to 0 as $\check{p}_0$ approaches the threshold p_f^*, so that the equilibrium rate of investment approaches infinity. The following theorem characterizes the unique symmetric equilibrium under public information.

Theorem 1 (Symmetric equilibrium with public information)**.** *There is a unique symmetric equilibrium. If $\check{p}_0 \geq p_f^*$, both agents invest immediately. If $p_l^* < \check{p}_0 < p_f^*$, each agent invests at constant rate $\beta^*(\check{p}_0)$ given by Equation (5) in the first phase, while the follower starts the project with delay $\tau^*(\check{p}_0)$. If $\check{p}_0 \leq p_l^*$, neither agent invests.*

3.2 Private information

We now turn to symmetric equilibria in the case in which agents' signals are private. We here focus on the parameter values that are relevant for our main result showing that welfare can be higher under asymmetric information.[4] Thus, we shall focus on the case $p_0 > p_l^*$ and a relatively uninformative signal, i.e., low ρ. The following theorem summarizes our findings for this case.

Theorem 2 (Symmetric equilibrium with private information)**.** *Let $p_0 > p_l^*$.*

(1.) *If $p_0 < p_f^*$, there exists a $\rho^* \in (1/2, 1)$ such that, for all $\rho \leq \rho^*$, there exists a symmetric equilibrium in which type g of each agent invests at rate μ_t^* given by Equation (14). Type b delays investment until t^*, given by Equation (15), and invests at constant rate $\beta^*(p_0(b,b))$ thereafter.*

(2.) *If $p_0 \geq p_f^*$, there exists a $\rho^{**} \in (1/2, 1)$ such that, for $\rho \leq \rho^{**}$, there exists a symmetric equilibrium in which type g invests immediately. If $p_0(b) \geq p_f^*$, type b invests immediately as well. Otherwise, type b of each agent invests immediately with some probability $\eta^* \in [0, 1]$, while investing at a random time arriving at rate $\beta^*(p_0(b,b))$ with probability $1 - \eta^*$.*

Thus, if $p_l^* < p_0 < p_f^*$ and ρ is low enough, good types face a war of attrition: they would prefer to invest immediately if their partner never invested, but they prefer to wait if the other agent invests immediately. Thus, in symmetric equilibrium, good types randomize, choosing their rate of investment precisely so as to render their partner of type g indifferent. Given that good types are indifferent, bad types strictly prefer not to invest. If $p_0 \geq p_f^*$ and ρ is low enough, investing is a dominant action for good types. Bad types, meanwhile, either invest immediately as well or do so with delay, thereby revealing their signal to be bad. In equilibrium, bad types may randomize between investing immediately and doing so with delay in such a way as to make their partner of type b indifferent between investing and waiting.

[4] Results for the other parameter values are available from the authors upon request.

4 Welfare and transparency

The following theorem characterizes the cooperative benchmark, which corresponds to a setting in which agents pool their private information and commit to a strategy at the outset of the game, seeking to maximize the sum of their payoffs. We shall see that, when signals are publicly observable, all equilibria are ex-post efficient when the prior belief that the state is good is either very high or very low.

Theorem 3 (Cooperative benchmark)**.** *In the cooperative benchmark, both agents invest immediately if $\check{p}_0 > \underline{p}$ and never invest if $\check{p}_0 < \underline{p}$. If $\check{p}_0 = \underline{p}$, either solution is optimal.*

As a comparison with the equilibrium in Theorem 2 shows, equilibrium under private information may be inefficient for intermediate prior beliefs. Inefficiencies arise because of leadership aversion: agents prefer being the follower and therefore wait if there is a sufficiently high probability that the other agent invests first. Free-riding aggravates its effect, because it lowers the value of being the leader, and thus reduces the rate at which the agents are willing to make the first investment.[5] It is therefore natural to ask if there is a social gain from increasing transparency. Naïve logic may suggest that more transparency should unambiguously lead to better outcomes, as it allows the agents to make better-informed decisions. This view, however, disregards the fact that agents wait inefficiently long for others to provide information for free when they are uncertain about the state, which negatively affects the efficiency of learning. Asymmetric information, as it turns out, can have positive side-effects. On the one hand, uncertainty introduces strategic risk about the other agent's future investment decisions, which lowers the incentives to delay initial investments. On the other hand, when signals are private, an early investment can signal good news, giving rise to an *encouragement-through-signalling effect*, by which an agent invests in order to motivate her partner to invest as well. There is thus a tradeoff between the gains from information aggregation and limiting public access to information in order to reduce the agents' incentives to free-ride.

We now state our main result, which formalizes this insight. It shows that private information is socially preferable if the prior belief that the state is good is high enough, and the signals are not too informative. Denote by $W_0(s^1, s^2)$ the expected social surplus generated in the unique symmetric equilibrium under public information, when $p_0(s_1, s_2)$ is the common initial belief that the state is good. Let $\tilde{W}$ denote the ex-ante expected social surplus generated in the symmetric equilibrium with private information described in 2. The following theorem shows that, for some parameters, $\tilde{W} \geq \mathbb{E}[W_0(s_1, s_2)]$.

Theorem 4 (Welfare improvement through private information)**.** *Let $p_0 > p_l^*$. There exists a $\rho^* > 1/2$ such that, for all signal precisions $\rho \in (1/2, \rho^*)$, we have $\tilde{W} \geq \mathbb{E}[W_0(s_1, s_2)]$. For*

[5] There is also a third type of inefficiency as agents may not invest in equilibrium although investment is socially optimal. With private signals, agents may also invest although investing is not ex-post socially optimal.

$p_l^* < p_0 < p_f^*$, *the inequality is strict.*

The condition on the signal precision ensures that it is socially optimal for both agents to invest immediately. Thus, the uncertainty only pertains to the strategic interaction. If $p_0 > p_f^*$, there always exists a range of signal precisions guaranteeing that both types invest immediately under private information, as in the efficient benchmark. If $p_0 < p_f^*$, then there are signal precisions such that the symmetric equilibrium has delayed entry. In this case, the welfare improvement results from strategic uncertainty and a spiraling effect, in which increasing pessimism about each other's types and increasing eagerness to invest result in ex-ante earlier investment.

5 Conclusion

We have investigated how private information affects agents' incentives to make irreversible investments. Agents benefit from their partner's experimentation because it provides them free information. However, delaying one's investment can signal bad news, thus discouraging experimentation by one's partner. We have shown that private information may increase social welfare in our setting by deterring delays in investments, which may be interpreted as signaling bad news.

One might wonder about the impact of allowing communication between agents in our setting. We conjecture that allowing for cheap talk between agents would not change the outcome. Indeed, if it did, either agent would always send the most positive signal to induce the other one to invest early. We should furthermore anticipate communication with verifiable messages to lead to unraveling, as agents would prefer to communicate their signal only if it is good, so that each agent can infer the other agent's signal perfectly. A deeper analysis of the role of communication in our setting would thus likely require that one abandon either our binary state space or the binary signal structure. We commend such an investigation to future research.

6 Appendix

Proof of Lemma 1. (i) That v_l is linear in p is obvious from its definition in Equation (1) and v_l is increasing in p because it follows from $\gamma c > y > 0$ that the second term in Equation (1) is negative. To see that v_l is decreasing in τ, note that $\lambda_2 < \lambda_1$, and hence

$$\frac{d}{d\tau} v_l(p,\tau) = -(r+\gamma)(1-p)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) < 0$$

for all $p \in (0,1)$ and $\tau \geq 0$. That v_l is convex in τ for all $p \in (0,1)$ follows from

$$\frac{d^2}{d^2\tau} v_l(p,\tau) = (r+\gamma)^2(1-p)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) > 0.$$

Finally, supermodularity holds because

$$\frac{d^2}{dp d\tau} v_l(p,\tau) = (r+\gamma)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) > 0.$$

(ii) Linearity of v_f in p is obvious from its definition in (2) and it is increasing in p because $\gamma c > y > 0$ implies that the first term in Equation (2) is positive and the second term is negative. For fixed $p \in (0,1)$, the derivative of v_f with respect to τ is

$$\frac{d}{d\tau} v_f(p,\tau) = e^{-r\tau}\left[(r+\gamma)e^{-\gamma\tau}(1-p)\lambda_2(c\gamma - y) - rpy\right].$$

Let $\hat{\tau}(p)$ be the (finite) solution to the first order condition $dv_f(p,\tau)/d\tau = 0$. The second term in brackets is positive, so that $dv_f(p,\tau)/d\tau > 0$ if $\tau < \hat{\tau}(p)$ and $dv_f(p,\tau)/d\tau < 0$ if $\tau > \hat{\tau}(p)$. Hence, v_f attains a global maximum at $\hat{\tau}(p)$. If $\hat{\tau}(p) \geq 0$, then solving

$$rpy + (r+\gamma)e^{-\gamma\hat{\tau}(p_0)}(1-p)(\lambda_2(y - c\gamma)) = 0$$

for $\hat{\tau}(p)$ shows that $\hat{\tau}(p) = \tau^*(p)$. If $\hat{\tau}(p) < 0$, then $v_f(p,\cdot)$ is strictly decreasing on $[0,\infty)$, and therefore assumes its maximum at 0.

(iii) If $p < p_f^*$ then $\tau^*(p) > 0$. Since v_l is strictly decreasing in τ and because $\tau^*(p)$ maximizes v_f, we have $v_f(p,\tau^*(p)) > v_f(p,0) = v_l(p,0) > v_l(p,\tau^*(p))$. If $p \geq p_f^*$, then $\tau^*(p) = 0$, and therefore $v_f(p,\tau^*(p)) = v_f(p,0) = v_l(p,0) = v_l(p,\tau^*(p))$. □

Proof of Lemma 2. From the definition of v_f, it follows that $v_f(p,0) > 0$ if and only if $p > \underline{p}$. Inspection reveals that $p_f^* > \underline{p}$ which implies $v_f(p_f^*,0) > 0$. Moreover, $v_l(p,0) = v_f(p,0)$ for all p, and therefore $v_l^*(p_f^*) = v_l(p_f^*,0) > 0$. Because v_l is continuous and strictly increasing, it follows from $v_l^*(p_l^*) = 0$ that $p_l^* < p_f^*$. Moreover, we have $0 = v_l^*(p_l^*) < v_l(p_l^*,0) = v_f(p_l^*,0)$. By Lemma 1, v_f is linearly increasing in its first argument. Since $v_f(\underline{p},0) = 0$, it follows from $v_f(p_l^*,0) > 0$ that

$\underline{p} < p_l^*$. □

Proof of Theorem 1. (i) Let $\check{p}_0 \geq p_f^*$. By definition of p_f^*, the follower's unique best response is to invest as soon as the other player has invested. It remains to be shown that there exists no equilibrium in which both players either never invest or invest (simultaneously) at some time $t > 0$. If a player deviates to investing at time 0, she would get a payoff of $w^* = \check{p}_0 y + (1-\check{p}_0)\lambda_2(y - \gamma c) > 0$, where the inequality holds because $\check{p}_0 \geq p_f^* > \frac{\lambda_2(\gamma c - y)}{y + \lambda_2(\gamma c - y)}$. In the conjectured equilibrium, by contrast, she obtains $e^{-rt}w^* < w^*$ ($t \in (0, \infty) \cup \{\infty\}$). Thus, she has an incentive to deviate and the only equilibrium is one in which both agents invest immediately.

(ii) Let $p_l^* < \check{p}_0 < p_f^*$. By definition of p_l^*, an agent would want to invest immediately if the other agent never invested. If the other agent invested, however, an agent would prefer not to invest, as this would give her $v_f^*(\check{p}_0)$ instead of $v_f(\check{p}_0, 0) < v_f^*(\check{p}_0)$, where the inequality holds because of $\check{p}_0 < p_f^*$ by definition of p_f^*. As shown in the main text, there is a unique investment rate $\beta^*(\check{p}_0)$ that renders the other agent indifferent between investing and not investing, which establishes the claim.

(iii) For $\check{p}_0 \leq p_l^*$, the claim follows immediately from the definition of p_l^*.

□

We need the following lemma for the proof of Theorem 2.

Lemma 3. *$v_f^*(p) - v_l^*(p)$ is a weakly decreasing function.*

Proof. Define $D(p, \tau) = v_f(p, \tau) - v_l(p, \tau)$. If $p \geq p_f^*$, then $\tau^*(p) = 0$ and therefore $D(p, \tau^*(p)) = 0$. Suppose $p < p_f^*$. Taking the derivative gives

$$\frac{dD(p, \tau^*(p))}{dp} = \frac{\partial D(p, \tau^*(p))}{\partial p} + \frac{\partial D(p, \tau^*(p))}{\partial \tau}\frac{d\tau^*(p)}{dp}. \tag{6}$$

Lemma 1 implies that the leader value is decreasing in τ, i.e., $dv_l(p, \tau^*(p))/d\tau < 0$, the optimal delay is decreasing, $d\tau^*(p)/dp < 0$, and that $\tau^*(p)$ maximizes the follower value, i.e., $dv_f(p, \tau^*(p))/d\tau = 0$. Therefore, $D(p)$ is strictly decreasing on $(0, p_f^*)$ if $\partial D(p, \tau^*(p))/\partial p < 0$. Using the definition of v_f in Equation (2), the derivative of the follower value is

$$\frac{\partial v_f(p, \tau^*(p))}{\partial p} = e^{-r\tau^*(p)}y + e^{-(r+\gamma)\tau^*(p)}\lambda_2(\gamma c - y).$$

From Equation (1), the derivative of the leader's value is

$$\frac{\partial v_l(p, \tau^*(p))}{\partial p} = y + (\lambda_1 + (\lambda_2 - \lambda_1)e^{-(r+\gamma)\tau^*(p)})(\gamma c - y).$$

The partial derivative of D with respect to p is

$$\frac{\partial D(p,\tau^*(p))}{\partial p} = -(1-e^{-r\tau^*(p)})y - \lambda_1(1-e^{-(r+\gamma)\tau^*(p)})(\gamma c - y) < 0.$$

□

Proof of Theorem 2. *Part (1).* Player i's expected value of becoming the leader in an equilibrium in which type b never invests is given by

$$V(q_t) = q_t v_l^*(p_0(g,g)) + (1-q_t)v_l^*(p_0),$$

where q_t is player i's equilibrium belief at time t that the other player $-i$ has observed a good signal. As $p_0(g,g) > p_0 > p_l^*$, $V(q_t) > 0$. Each agent is willing to randomize if she is indifferent between investing immediately and waiting for another instant. Hence, the value function for type g of the agent must satisfy the indifference condition

$$V(q_t) = \mu_t q_t v_f(p_0(g,g))dt + (1 - rdt - \mu_t q_t dt)V(q_{t+dt}). \tag{7}$$

A first-order Taylor approximation of the indifference condition (7) yields

$$V(q_{t+dt}) = V(q_t) + dV(q_t)dq_t + o(dt), \tag{8}$$

where by definition of V, we have $dV(q_t)/dq_t = v_l^*(p_0(g,g)) - v_l^*(p_0)$. Bayes' rule implies that the posterior belief at $t+dt$ is

$$q_{t+dt} = \frac{q_t(1-\mu_t dt)}{1-q_t\mu_t dt}.$$

The differential change in belief is therefore

$$\frac{dq_t}{dt} \equiv \lim_{dt\to 0}\frac{q_{t+dt}-q_t}{dt} = -\mu_t q_t(1-q_t). \tag{9}$$

If we now substitute equations (8) and (9) in the indifference condition (7) and ignore higher order terms, we obtain the expression

$$r\left[q_t v_l^*(p_0(g,g)) + (1-q_t)v_l^*(p_0)\right] = \mu_t q_t\left[v_f^*(p_0(g,g)) - v_l^*(p_0(g,g))\right]. \tag{10}$$

We choose $\rho^* \in (1/2, 1)$ such that $p_0(g,g) < p_f^*$ and hence the right-hand side of this equation is strictly positive. (Such a $\rho^* \in (1/2,1)$ exists since $p_0 < p_f^*$.) Simplifying and solving the equation for μ_t yields

$$\mu_t = \frac{r\left[q_t v_l^*(p_0(g,g)) + (1-q_t)v_l^*(p_0)\right]}{q_t\left[v_f^*(p_0(g,g)) - v_l^*(p_0(g,g))\right]}. \tag{11}$$

Here, μ_t is the rate of investment for type g of each agent in the symmetric equilibrium at a given belief q_t. Substituting this last expression into Equation (9), we obtain the evolution of the posterior q_t in equilibrium:

$$dq_t = -rq_t(1-q_t)\frac{q_t v_l^*(p_0(g,g)) + (1-q_t)v_l^*(p_0)}{q_t\left[v_f^*(p_0(g,g)) - v_l^*(p_0(g,g))\right]}dt. \tag{12}$$

We obtain the equilibrium belief and equilibrium investment rate at each time t by solving Equation (12) with given initial belief q_0. Setting $\mathbb{E}[v_l^*|g] = q_0 v_l^*(p_0(g,g)) + (1-q_0)v_l^*(p_0)$, the initial value problem (12) has the unique solution

$$q_t^* = \frac{e^{-t\beta^*(p_0(g,g))}\mathbb{E}[v_l^*|g] - q_0(g)v_l^*(p_0)}{q_0(g)\left[v_l^*(p_0(g,g)) - v_l^*(p_0)\right] + e^{-t\beta^*(p_0(g,g))}\mathbb{E}[v_l^*|g]}. \tag{13}$$

We now substitute q_t^* into Equation (11) and simplify to obtain the equilibrium rate of investment

$$\mu_t^* = \frac{e^{-t\beta^*(p_0(g,g))}\mathbb{E}[v_l^*|g]}{e^{-t\beta^*(p_0(g,g))}\mathbb{E}[v_l^*|g] - p_0 v_l^*(p_0)}\beta^*(p_0(g,g)). \tag{14}$$

The investment rate $\mu_t^* \in (0,\infty)$ for $t < t^*$, and diverges to $+\infty$ as $t \to t^*$, where

$$t^* = \log\left(1 + \frac{p_0(g,g)v_l^*(p_0(g,g))}{p_0 v_l^*(p_0)}\right)^{\beta^*(p_0(g,g))}. \tag{15}$$

It remains to be shown that it is optimal for type b of each agent to delay investment indefinitely. Note that $p_0(b,b) < p_0 < p_0(g,g)$, and v_l is strictly increasing in its first argument by Lemma 1. Moreover, the difference $v_f^* - v_l^*$ is weakly decreasing by Lemma 3. Thus Equation (10) implies

$$q_t(b)v_l(p_0, \tau^*(p_0(g,g))) + (1-q_t(b))v_l^*(p_0(b,b), \tau^*(p_0)) < \mu_t^* q_t(b)[v_f^*(p_0) - v_l^*(p_0)] \tag{16}$$

This left hand-side is the expected payoff of an agent with signal b from investing immediately, the right-hand side is the payoff gain from delaying investment. Thus, agents with a bad signal strictly prefer to delay investment.

Part (2). Let $p_0 \geq p_f^*$. Suppose that, at $t = 0$, agents with signal g invest with probability 1, while agents with signal b invest with probability 1 if $p_0(b) \geq p_f^*$ and with some probability

$\eta \in [0,1]$ otherwise. If no agent has invested at time $t = 0$, agents invest at rate $\beta(p_0(b,b))$ if $p_0(b,b) < p_f^*$; otherwise, they invest immediately.[6] With this pair of strategies, the first phase either never continues past $t = 0$ (if $\eta = 1$), or continues past $t = 0$ only if neither agent invested (if $\eta < 1$), and thus in equilibrium, at any $t > 0$, it is common knowledge that both agents have a bad signal.

Denote by $V_{1s}(\eta)$ the value of investing at $t = 0$ for an agent with signal s when the other one uses the previous strategy, and let $V_{0s}(\eta)$ be the respective value of waiting at $t = 0$. These functions are convex combinations of continuous functions and hence continuous. Under Bayesian updating, a deviation may lead to inconsistencies in beliefs. Specifically, if an agent chooses not to invest despite observing signal g, then the other agent will falsely conjecture that she observed signal b, and start to play the symmetric equilibrium under common belief $p_0(b,b)$. The agent with signal g then invests immediately.[7]

Note that $V_{0b}(1) \geq V_{1b}(1)$, since bad types always have weak incentives to wait when the other agent invests with probability one. Furthermore, $V_{0b}(1) = V_{1b}(1)$ if and only if $p_0(b) \geq p_f^*$. There are thus two cases to consider, $V_{0b}(0) < V_{1b}(0)$ and $V_{0b}(0) > V_{1b}(0)$.

1. Suppose $V_{0b}(0) < V_{1b}(0)$, i.e., an agent with a bad signal prefers to invest immediately if the other agent invests with zero probability after a bad signal. In this case, there exists a partial (or full) pooling equilibrium in which type g always invests while type b invests with probability $\eta^* \in (0,1]$. Indeed, by continuity of V_{0b} and V_{1b}, there exists an $\eta^* \in (0,1]$ such that $V_{0b}(\eta^*) = V_{1b}(\eta^*)$, so that an agent of type b is indifferent between investing and not investing, given the other agent invests with probability η^* after observing signal b. We shall now verify an agent of type g's incentives to invest. By indifference, we have

$$
\begin{aligned}
V_{1b}(\eta^*) - V_{0b}(\eta^*) &= (q_0(b) + (1 - q_0(b))\eta^*)\big(v_l(\tilde{p}(\eta^*, b)), 0) - v_f^*(\tilde{p}(\eta^*, b))\big) \\
&\quad + (1 - q_0(b))(1 - \eta^*)\big(v_l^*(p_0(b,b), \tau^*(\tilde{p}(\eta^*, b))) - \max\{0, v_l^*(p_0(b,b))\}\big) = 0,
\end{aligned}
$$

where $\tilde{p}(\eta, b)$ denotes a player's updated belief about the state of the world, given that she has observed a signal of b and that her partner, whom she expects to invest with probability 1 (η) upon observing signal g (b), has invested. As $p_0 > p_l^*$, there exists $\rho^{**} \in (1/2, 1)$ such that, for all $\rho \in (1/2, \rho^{**})$, $p_0(b,b) > p_l^*$, and hence

$$
v_l(p_0(b,b), \tau^*(\tilde{p}(\eta^*, b))) \geq \max\{0, v_l^*(p_0(b,b))\}
$$

Since v_l is strictly increasing in its first argument, it follows that $v_l(p_0, \tau^*(\tilde{p}(\eta^*, b))) > 0$. Now,

[6] If $\eta = 1$, this implies that, off the path of play, agents believe that an agent who did not invest at $t = 0$ must have received a bad signal for sure. Please also see our remark below.

[7] There is a minor technical issue here regarding the timing of actions. The described behavior requires sequential moves at $t = 0$, which we do not allow. For simplicity, we use a reduced-form representation, writing payoffs in a way that reflects this behavior.

we have

$$V_{1g}(\eta^*) - V_{0g}(\eta^*) = (q_0(g) + (1 - q_0(g))\eta^*)\big(v_l(\tilde{p}(\eta^*, g), 0) - v_f^*(\tilde{p}(\eta^*, g))\big) \\ + (1 - q_0(g))(1 - \eta^*)\big(v_l(p_0, \tau^*(\tilde{p}(\eta^*, b))) - \max\{0, v_l(p_0, \tau^*(p_0(b, b))\}\big).$$

Since $\tilde{p}(\eta^*, g) \geq p_0 \geq p_f^*$, we have $\tau^*(\tilde{p}(\eta^*, g)) = 0$, and thus $v_l(\tilde{p}(\eta^*, g), 0) - v_f^*(\tilde{p}(\eta^*, g)) = 0$. Finally, since $v_l(p_0, \tau^*(\tilde{p}(\eta^*, b))) \geq v_l(p_0, \tau^*(p_0(b, b))$ we have $V_{1g}(\eta^*) - V_{0g}(\eta^*) \geq 0$, so that it is a best response for an agent with signal g to invest.

2. Now, suppose $V_{0b}(0) \geq V_{1b}(0)$. Then,

$$V_{1g}(0) - V_{0g}(0) = q_0(g)\big(v_l(p_0(g, g), 0) - v_f^*(p_0(g, g))\big) \\ + (1 - q_0(g))\big(v_l^*(p_0) - \max\{0, v_l(p_0, \tau^*(p_0(b, b))\}\big)$$

Since $p_0(g, g) > p_0 \geq p_f^*$, we have $v_l(p_0(g, g), 0) - v_f^*(p_0(g, g)) = 0$, and, from $p_0 > p_0(b, b) > p_l^*$, it follows $v_l^*(p_0) - \max\{0, v_l(p_0, \tau^*(p_0(b, b))\} \geq 0$, so that

$$V_{1g}(0) - V_{0g}(0) \geq 0.$$

Thus, in this case, there exists a fully separating equilibrium in which g-types invest at $t = 0$, whereas b-types do not.

□

Proof of Theorem 3. The team's objective is to maximize

$$w(p, \tau) = \max\left\{0, v_l(p, \tau) + v_f(p, \tau)\right\},$$

where τ denotes the delay with which the follower invests. Using Equations (1) and (2), we can write explicitly:

$$v_l(p, \tau) + v_f(p, \tau) = (1 + e^{-r\tau})py + (1 - p)\left[\lambda_1 + (2\lambda_2 - \lambda_1)e^{-(r+\gamma)\tau}\right](y - \gamma c).$$

It follows from the definitions of λ_1 and λ_2 that $2\lambda_2 - \lambda_1 = \lambda_1\lambda_2$. Therefore, the marginal value of delaying the second investment is

$$\frac{\partial(v_l + v_f)(p, \tau)}{\partial\tau} = -re^{-r\tau}[py + (1 - p)\lambda_2(y - \gamma c)e^{-\gamma\tau}]. \tag{17}$$

The expression in brackets is strictly increasing in τ. This implies that $\partial(v_l + v_f)(p, \tau)/\partial\tau < 0$ for all $\tau \geq 0$ whenever $p > \underline{p}$, where

$$\underline{p} := \frac{\lambda_2(\gamma c - y)}{y + \lambda_2(\gamma c - y)}, \tag{18}$$

in which case it is socially optimal to make the second investment immediately. If $p \leq \underline{p}$, then the socially optimal delay solves the first-order condition $dw(p,\tau)/d\tau = 0$. Thus, the optimal delay is given by

$$\tau^s(p) = \begin{cases} [\phi(\underline{p}) - \phi(p)]/\gamma & \text{if } p < \underline{p}, \\ 0 & \text{if } p \geq \underline{p}. \end{cases}$$

It remains to be shown that, if $p \leq \underline{p}$, $w(p, \tau^s(p)) = 0$. Indeed,

$$w(p,\tau^s(p)) = \max\left\{py + (1-p)\lambda_1(y - \gamma c) + py\left(1 - \frac{\lambda_1}{1-p}\right)\left(\frac{p(1-\underline{p})}{\underline{p}(1-p)}\right)^{\frac{r}{\gamma}}, 0\right\}.$$

Since $p \leq \underline{p}$, it is sufficient to show that $py + (1-p)\lambda_1(y-\gamma c) + py\,(1 - \lambda_1/(1-p)) \leq 0$ and $py + (1-p)\lambda_1(y - \gamma c) \leq 0$. As both inequalities are satisfied for $p \leq \underline{p}$, this concludes the proof. □

Proof of Theorem 4. Let P be the distribution over signals for given parameters p_0 and ρ; i.e., we write $P(s)$ for the probability that a given agent's signal is s and $P(s_1, s_2)$ for the probability that the pair of signals is (s_1, s_2).

(1.) Suppose $p_0 > p_f^*$. Then, choose $\rho^* > 1/2$ such that $p_0(b) > p_f^*$ and $p_0(b,b) > \underline{p}$. By Theorem 2, it follows that for all $\rho < \rho^*$, there exists a pooling equilibrium in which each type of each agent invests immediately. By Theorem 3, this equilibrium is efficient. Hence $\tilde{W} \geq E[W(s_1, s_2)]$.

(2.) Let $p_f^* > p_0 > p_l^*$. Choose $\rho^* > 1/2$ such that $p_f^* > p_0(g,g)$ and $p_0(b,b) \geq \underline{p}$. By Theorem 2, there exists an equilibrium with delayed entry. It follows from arguments in the proof of Theorem 2 that the expected equilibrium value of the good type of each agent is $\mathbb{E}[v_l^*|g]$. By Equation (16), bad types strictly prefer to delay investment at each $t < t^*$. The expected payoff for an agent of type b who deviates by investing before time t^* is given by

$$q_t(b)v_l(p_0, \tau^*(p_0(g,g))) + (1 - q_t(b))v_l^*(p_0(b,b), \tau^*(p_0)) > \mathbb{E}[v_l^*|b].$$

Therefore, the expected social surplus for each agent is

$$\tilde{W} > P(g)\mathbb{E}[v_l^*|g] + P(b)\mathbb{E}[v_l^*|b] = \mathbb{E}[W(s^1, s^2)].$$

(3.) Let $p_0 = p_f^*$. It follows from Theorem 2 that good types invest immediately with probability one, and bad types invest immediately with some probability $\eta^* \in [0,1]$. Define $w(\eta) =$

$P(g)V_{1g}(\eta) + P(b)V_{1b}(\eta)$, where V_{1s} again denotes the value of investing immediately for an agent with signal s. Since bad types must be indifferent between investing immediately and waiting, we have $\tilde{W} = w(\eta^*)$. We show that there exists a $\rho^* > 1/2$ such that $w(\eta) > E[W]$ for all $\rho \in (1/2, \rho^*)$ and $\eta \in [0,1]$. Note that $P(g)q_0(g) = P(g,g)$, $P(g)(1-q_0(g)) = P(b)q_0(b) = P(b,g)$ and $P(b)(1-q_0(b)) = P(b,b)$. Using $q_0(b)p_0 + (1-q_0(b))p_0(b,b) = p_0(b)$ together with the linearity of $v_l(p,0)$, we can write $w(\eta)$ as:

$$w(\eta) = P(g,g)v_l(p_0(g,g),0) + P(b,g)v_l(p_0,0) \\ + (P(g,b)+P(b,b))\Big[\eta v_l(p_0(b),0) + (1-\eta)v_l(p_0(b),\tau^*(\tilde{p}(\eta,b))\Big].$$

When signals are public, then, after each realized pair of signals resulting in the posterior belief $\check{p}_0$, each agent's equilibrium payoff is $v_l^*(\check{p}_0)$. Thus, the expected welfare under public information can be written as

$$E[W] = P(g,g)v_l^*(p_0(g,g)) + P(b,g)v_l^*(p_0) \\ + (P(g,b)+P(b,b))\Big[q_0(b)v_l^*(p_0) + (1-q_0(b))v_l^*(p_0(b,b))\Big]. \quad (19)$$

where we use the fact that $q_0(b) = P(g,b)/(P((g,b) + P(b,b))$. By the definition of τ^* in Lemma 1, we have

$$v_l^*(p) = v_l(p,\tau^*(p)) = py + (1-p)\lambda_1(y-\gamma c) + (1-p)e^{-(r+\gamma)\tau^*(p)}(\lambda_2-\lambda_1)(y-\gamma c).$$

Since $p_0 = p_f^*$, we have $\tau^*(p_0) = \tau^*(p_0(g,g)) = 0$. Thus, we have that $w(\eta) > E[W]$ if and only if

$$\eta v_l(p_0(b),0)+(1-\eta)v_l(p_0(b),\tau^*(\tilde{p}(\eta,b))) > q_0(b)v_l(p_0,0)+(1-q_0(b))v_l^*(p_0(b,b)). \quad (20)$$

We define $\psi(p) := \frac{p}{1-p} = e^{\phi(p)}$ and $\alpha := \frac{r+\gamma}{\gamma} > 1$. Then, the left-hand side of Inequality (20) can be written as

$$\eta v_l(p_0(b),0) + (1-\eta)v_l(p_0(b),\tau^*(\tilde{p}(\eta,b)) = p_0(b)y + (1-p_0(b))\lambda_1(y-\gamma c) \\ + (1-p_0(b))\left[\eta + (1-\eta)\psi(\tilde{p}(\eta,b))^{\alpha}\psi(p_f^*)^{-\alpha}\right](\lambda_2-\lambda_1)(y-\gamma c). \quad (21)$$

From Bayes' rule and the definition of ψ, we have

$$\psi(p_0(b)) = \frac{p_0}{1-p_0}\frac{1-\rho}{\rho} = \psi(p_0)/\psi(\rho), \quad \psi(\tilde{p}(\eta,b)) = \psi(p_0(b))\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right).$$

If we now use the previous equalities to factor out $\psi(p_0(b))^\alpha \psi(p_f^*)^{-\alpha}$ from the square brackets in (21), we obtain

$$\eta v_l(p_0(b),0) + (1-\eta) v_l(p_0(b), \tau^*(\tilde{p}(\eta,b)) = p_0(b)y + (1-p_0(b))\lambda_1(y-\gamma c)$$
$$+ (1-p_0(b))\psi(p_0(b))^\alpha \psi(p_f^*)^{-\alpha}\left[\eta\psi(\rho)^\alpha + (1-\eta)\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right)^\alpha\right](\lambda_2-\lambda_1)(y-\gamma c). \quad (22)$$

The right-hand side of Inequality (20) is given by

$$q_0(b)v_l(p_0,0) + (1-q_0(b))v_l^*(p_0(b,b)) = p_0(b)y + (1-p_0(b))\lambda_1(y-\gamma c)$$
$$+\left[q_0(b)(1-p_0) + (1-q_0(b))(1-p_0(b,b))\psi(p_0(b,b))^\alpha\psi(p_f^*)^{-\alpha}\right](\lambda_2-\lambda_1)(y-\gamma c). \quad (23)$$

From Bayes' rule it follows that

$$\frac{q_0(b)}{1-p_0(b)} = \frac{p_0(b)\rho + (1-p_0(b))(1-\rho)}{1-p_0(b)} = \left(\frac{p_0}{1-p_0}\frac{1-\rho}{\rho}\right)\rho + (1-\rho) = \frac{1-\rho}{1-p_0},$$
$$\frac{1-q_0(b)}{1-p_0(b)} = \frac{p_0(b)(1-\rho) + (1-p_0(b))\rho}{1-p_0(b)} = \left(\frac{p_0}{1-p_0}\frac{1-\rho}{\rho}\right)(1-\rho) + \rho = \frac{\rho}{1-p_0(b,b)}.$$

Using these equalities together with the identity

$$\psi(p_0(b,b)) = \frac{p_0(b)}{1-p_0(b)}\frac{1-\rho}{\rho} = \psi(p_0(b))\psi(1-\rho)$$

to factor out $(1-p_0(b))\psi(p_0(b))^\alpha\psi(p_f^*)^{-\alpha}$ from the square brackets in (23), we obtain

$$q_0(b)v_l(p_0,0) + (1-q_0(b))v_l^*(p_0(b,b)) = p_0(b)y + (1-p_0(b))\lambda_1(y-\gamma c)$$
$$+ (1-p_0(b))\psi(p_0(b))^\alpha\psi(p_f^*)^{-\alpha}\left[(1-\rho)\psi(\rho)^\alpha + \rho\psi(\rho)^{-\alpha}\right](\lambda_2-\lambda_1)(y-\gamma c).$$

Define the functions

$$h(\eta,\rho) := \eta\psi(\rho)^\alpha + (1-\eta)\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right)^\alpha, \quad g(\rho) := (1-\rho)\psi(\rho)^\alpha + \rho\psi(\rho)^{-\alpha}.$$

Condition (20) is thus equivalent to $\inf_\eta h(\eta,\rho) > g(\rho)$. One calculates that the partial derivative of h at $\rho = 1/2$ is $\lim_{\rho\to 1/2}\partial_\rho h(\eta,\rho) = 4\alpha\left(2\eta^2 - \eta + 1\right)/(\eta+1)$. The function $\lim_{\rho\to 1/2}\partial_\rho h(\eta,\rho)$ has its minimum in η at $\sqrt{2}-1$ and is thus larger than $4\left(4\sqrt{2}-5\right) > 0$. On the other hand $g'(1/2) = 0$. Thus, there exists a $\rho^* > 1/2$ such that for all $\rho \in (1/2, \rho^*)$, we have $\tilde{W} = w(\eta^*) > E[W]$.

□

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