Relational Contracts with Private Information: The Upside of Implicit Downsizing Costs

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Private Information in Relational Contracts

Introduction

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Benchmark: Public Info

Private Types

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Literature
A principal wants to give an agent incentives to exert effort repeatedly; has some private info about productivity of agent’s labour. Optimal effort depends on this productivity.
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Only one-period (formal) contracts; principal can pay the agent a “voluntary” bonus to reward him for his effort.

Bonus is bounded above by value of \textit{future} relationship.
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Novelty: When deciding on the bonus payment, the principal has private information about the productivity of the agent’s effort in the next period.
Model

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- Setup
- Timing
- Objectives
- PPE

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One principal, one agent (both risk neutral).
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Labour productivity in period $t$ depends on type $\theta_t \in \{\theta^l, \theta^h\}$ ($0 < \theta^l < \theta^h$).
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Labour productivity in period \( t \) depends on type \( \theta_t \in \{ \theta^l, \theta^h \} \) (\( 0 < \theta^l < \theta^h \)).

\[ \theta_1 = \theta^h; \theta_t = \theta^h \text{ with probability } q \in (0, 1) \text{ for all } t = 2, 3, \ldots \text{ (iid).} \]
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5. Output $y_t = g(n_t)$ is realized and publicly observed (not contractible!); $g: \mathbb{R}_+ \to \mathbb{R}_+$ is $C^2$, with $g(0) = 0$, $g' > 0 > g''$, $g'(0) = \infty$, $g'(\infty) = 0$; profit $\theta_t y_t$.
   
   $\rightarrow$ First-best effort $n^*(\theta)$ given by $\theta g'(n^*(\theta)) = c.$
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6. Bonus $b_t \geq 0$ is paid by the P to A. P sends A cheap-talk message.
The Players’ Payoffs

Principal:

\[
d_t \left( \theta_t g(n_t) - w_t \right) + E \left[ -b_t + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} d_\tau \left( \theta_\tau g(n_\tau) - w_\tau - b_\tau \right) \right].
\]
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Agent:

\[ d_t (w_t - c n_t) + E \left[ b_t + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} d_\tau (-c n_\tau + w_\tau + b_\tau) \right]. \]
Solution Concept

Solution Concept: PPE (standard in this literature).

Public strategy = Strategy which does not condition on past private info (which is not payoff-relevant!).
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⇒ On-path equilibrium actions completely determined by past type realizations $\theta^t$.

Look for a best PPE for the principal. This equilibrium also maximizes joint surplus.
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- \( \theta \) public info
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The Firm’s Type is Public Information: Constraints

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The Firm’s Type is Public Information: Constraints

1. Agent needs to accept offer: $U(\theta^t) \geq 0$ for all $\theta^t$. 
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2. After receiving \( w_t \), agent must find it optimal to exert the right level of effort:

\[
-n(\theta^t)c + q\left(b^h(\theta^t) + \delta U^h(\theta^t)\right) + (1 - q)\left(b^l(\theta^t) + \delta U^l(\theta^t)\right) \\
\geq -\tilde{n}c + q\left(b^h(\theta^t, \tilde{n}) + \delta U^h(\theta^t, \tilde{n})\right) \\
+ (1 - q)\left(b^l(\theta^t, \tilde{n}) + \delta U^l(\theta^t, \tilde{n})\right).
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The Firm’s Type is Public Information: Constraints

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3. It must be optimal for the principal to make equilibrium bonus payments

$$-b^h(\theta^t) + \delta \Pi^h(\theta^t) \geq 0 \quad \text{(DEh)}$$
$$-b^l(\theta^t) + \delta \Pi^l(\theta^t) \geq 0. \quad \text{(DEl)}$$
(DEh) and (DEI) can be combined into

\[- (q b^h(\theta^t) + (1 - q) b^l(\theta^t)) + \delta (q \Pi^h(\theta^t) + (1 - q) \Pi^l(\theta^t)) \geq 0. \]

(DE)
Equilibrium effort only depends on the current state: 
\[ n(\theta^t) = n(\theta^t) \] 

Only observable deviations; no need to destroy surplus on the equilibrium path \( \Rightarrow \) Want to be as close to FB-level as possible

Stationary environment (iid): Maximum enforceable effort levels the same for every history \( \theta^t \).
Profit-Maximizing Equilibrium with Public Info

**Proposition:** Assume the firm’s type is publicly observable. Then, there are levels of the discount factor, $\delta$ and $\bar{\delta}$, with $0 < \underline{\delta} < \delta < \bar{\delta} < 1$, such that

- $n^h$ and $n^l$ are at their efficient levels for $\delta \geq \bar{\delta}$.
- $n^h \geq n^l$, but $n^h$ is inefficiently low, and $n^l$ is at its efficient level for $\underline{\delta} \leq \delta < \bar{\delta}$;
- $n^h = n^l$, and both effort levels are inefficiently low for $\delta < \underline{\delta}$. 
Principal needs incentives not to misrepresent his private type after any history $\theta_t$:

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\[-b^h(\theta^t) + \delta \Pi^h(\theta^t) \geq -b^l(\theta^t) + \delta \tilde{\Pi}^l(\theta^t) \quad (TTh)\]

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where

$$\tilde{\Pi}^l(\theta^t) = \Pi^l(\theta^t) + \theta^h g(n^l(\theta^t)) - \theta^l g(n^l(\theta^t));$$

$$\tilde{\Pi}^h(\theta^t) = \Pi^h(\theta^t) - \theta^h g(n^h(\theta^t)) + \theta^l g(n^h(\theta^t)).$$

Uses One-Deviation Principle.
Overview of Constraints

\[ U(\theta^t) \geq 0 \]  \hspace{2cm} \text{(IR)}

\[-n(\theta^t)c + q\left(b^h(\theta^t) + \delta U^h(\theta^t)\right) + (1-q)\left(b^l(\theta^t) + \delta U^l(\theta^t)\right) \geq 0 \]  \hspace{2cm} \text{(IC)}

\[-b^h(\theta^t) + \delta \Pi^h(\theta^t) \geq 0 \]  \hspace{2cm} \text{(DEh)}

\[-b^l(\theta^t) + \delta \Pi^l(\theta^t) \geq 0. \]  \hspace{2cm} \text{(DEl)}

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The (EC) Constraint

Agency problem with private info boils down to constraint

\[-n(\theta^t) c + \delta q \Pi^h(\theta^t) + \delta (1-q) \Pi^l(\theta^t) \geq \delta q g(n^l(\theta^t))(\theta^h - \theta^l).\]

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The (EC) Constraint

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(EC)

(LHS) like (DE) constraint

(RHS) New effect: Information Rent of the P, who always has the option of claiming tomorrow’s profits are lower (only \(\theta^l g(n^l(\theta^t))\)) than they actually are (\(\theta^h g(n^l(\theta^t))\)).
**Lemma:** There exists an optimal equilibrium with the property that, for every two histories $\theta^t$ and $\tilde{\theta}^t$, $n^h(\theta^t) = n^h(\tilde{\theta}^t)$. Furthermore, for every history $\theta^t$, $n^l(\theta^t) = n^l_i$, where $i \in \{0, 1, 2, \ldots\}$ denotes the number of previous consecutive periods $\tau$ with $\theta_\tau = \theta^l$. 
Lemma: There exists an optimal equilibrium with the property that, for every two histories \( \theta_t \) and \( \tilde{\theta}_t \), \( n^h(\theta_t) = n^h(\tilde{\theta}_t) \). Furthermore, for every history \( \theta_t \), \( n^l(\theta_t) = n^l_i \), where \( i \in \{0, 1, 2, \ldots\} \) denotes the number of previous consecutive periods \( \tau \) with \( \theta_\tau = \theta^l \).

\( n^h \) only enters the (LHS) of the (EC) constraint; reduction of \( n^h(\theta_t) \) does not increase \( P \)'s commitment. \( \Rightarrow \) Have the \( n^h \) that is the closest possible to the FB after any history \( \theta_t \).

Environment stationary \( \Rightarrow \) Closest \( n^h \) to the FB possible is the same after any history \( \theta_t \).
Dynamics of Equilibrium Employment

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Environment stationary $\Rightarrow$ Closest $n^h$ to the FB possible is the same after any history $\theta^t$.

By contrast, reduction of $n^l$ enhances P’s commitment.
Result: High $\delta$

**Proposition:** There exists a $\delta \in (0, 1)$ such that optimal equilibrium profits are equal to first-best surplus for all $\delta > \bar{\delta}$. In this case, for every history $\theta^t$, first-best effort levels $n^*(\theta^t)$ can be implemented.
**Intermediate $\delta$**

**Proposition:** There exist discount factors $\underline{\delta}$ and $\overline{\delta}$, with $0 < \underline{\delta} < \overline{\delta} < 1$, such that, in an optimal equilibrium, for $\delta \in (\underline{\delta}, \overline{\delta})$, $n^h$ and $n^l_0$ are inefficiently low; for all $i \geq 1$, $n^l_i = n^\ast(\theta^l)$. 
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$(ECh)$ binds; need to reduce $n^h$.

$n^l_0$ is also reduced! $\Rightarrow$ Cost of not telling the truth in high state goes up; “transferring effort from low to high state”

$n^l_i$ at FB-levels! Discount factor is still high enough for $n^* (\theta^l)$ to be enforceable.

Optimal effort in low periods immediately following a high period is not sequentially optimal.

“Differential punishment of on-path and off-path principal”
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$\Rightarrow$ Implicit Downsizing Costs
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  - Reduction of labour input not sequentially optimal!
  - On-path destruction of surplus (even though private info is one-sided)
• Bull (1987); MacLeod & Malcomson (1989)
• Levin (2003)
• Halac (2012): P has private info about his (persistent) outside option.
• Li & Matouschek (2013): P has private information about cost of compensating the agent.
• Malcomson (2015): P has private info about the value of A’s effort in the current period; A has private info about costs.
• Malcomson (2016): A’s persistent cost type is private information; full separation not possible when continuation payoffs are on the Pareto frontier.