Strategic investment and learning with private information*

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July 27, 2022

Abstract

We study social learning from actions and outcomes. Agents learn about future returns through privately observed signals, others' investment decisions and public experimentation outcomes when returns are realized. We characterize symmetric equilibria, and relate the extent of strategic delay of investments in equilibrium to the primitives of the information structure. Agents invest without delay in equilibrium when the most optimistic interim belief exceeds a threshold. Otherwise, delay in investments induces a learning feedback that may either raise or depress beliefs and investment choices. We show that, although ours is a strategic-experimentation game of pure informational externalities, private information may increase ex-ante welfare.

Keywords— Social learning, investment timing, strategic delay, experimentation, signaling

*We owe thanks for comments and suggestions to an anonymous associate editor and an anonymous referee, as well as Matthias Fahn, Raphaël Lévy, Sven Rady, and audiences at Universidad Diego Portales, Lemma Paris-II Panthéon-Assas, Université Paris-Dauphine, the 2018 Stony Brook Workshop on Stopping Time Problems in Game Theory, the 2019 Annual Meeting of the Society for the Advancement of Economic Theory, the 2021 CREED/ TI Workshop on Learning: Experimentation and Information Design, the 2021 Workshop on *Incentives and dynamic games*, Lemma Université Paris-II, the 7th Workshop on Stochastic Methods in Game Theory (Erice, 2022), the Paris Game-Theory Seminar. Peter Wagner gratefully acknowledges funding from the Hausdorff Center for Mathematics. Nicolas Klein gratefully acknowledges support from the Social Sciences and Humanities Research Council of Canada and the Deutscher Akademischer Austauschdienst (DAAD).

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1 Introduction

Learning from peer experience is an important contributor to the proliferation of innovative technology, thus promoting economic development and growth. That observational learning plays a crucial role in the diffusion of innovation is empirically well-documented. Health professionals learn about medical innovations from the experience of their colleagues (Becker, 1970), households learn about new consumer products from friends and neighbors (Liu et al., 2014; Goolsbee et al., 2002), farmers learn about the qualities of new types of crop from the performance of their peers (Conley and Udry, 2010) and law-makers take into account the experience with legislation in other countries (Aidt and Jensen, 2009). The economics literature typically assumes that agents learn exclusively from the behavior of others, or exclusively from their own, and others' experiences. In reality, however, learning through observation often involves both channels. Think for instance of a farmer who updates his belief about a new type of crop by observing both the adoption behavior of a neighboring farmer and his yield. A physician may likewise learn about the effectiveness of a new drug both by observing that her colleagues prescribe it, and from the health outcomes of their patients.

In this paper, we study how such peer learning effects influence the timing of investments in new technologies. We consider a dynamic investment model in which two agents are privately informed about the value of a new technology, and they must choose whether and when to invest in adopting it. Adopting the technology generates a positive return, but there is also the possibility of a disastrous failure that may occur at an unknown time. Delaying the investment allows agents to observe each other's actions and experiences, thereby acquiring additional information without facing the risk of failure. There are two crucial features that characterize the interaction between agents. First, learning from actions *and* outcomes creates a signaling motive. By behaving in a way that signals good news about the technology, agents can encourage one another to invest early, and profit from observing one's opponent's returns. Second, there are no payoff externalities, that is, one agent's investment decision does not directly affect the other's payoff. We furthermore deliberately restrict attention to the case of substantial investment costs, in order to rule out the use of sophisticated punishment strategies, thus allowing us to focus on the effects resulting from information spill-overs. We address the following questions in this setup: How do these learning spillovers influence the agents' incentives to invest and how are these incentives affected by the strength of private beliefs? Under what conditions is private information revealed in equilibrium and is it revealed instantly or over time? Does private information increase or decrease efficiency compared to a scenario in which all information is public?

We show that information asymmetry among agents can increase investment efficiency. It is well known that informational spill-overs may result in inefficiently late investments. In settings in which agents learn through others' actions, they have an incentive to wait to observe what others do, which may lead to herding, and a failure of information aggregation (Chamley and Gale, 1994). When agents learn exclusively by observing others' payoffs, they have an incentive to wait for others to invest first, and then learn about the benefits of the investment from their experience. When agents learn from actions and payoffs under asymmetric information, one might expect these two forms of inefficiency to reinforce each other. However, we show that, when agents learn from others' actions and payoffs, these inefficiencies are mitigated.

To show this, we compare the surplus generated in an equilibrium with private information to that of a benchmark scenario in which the players are assumed to share their information truthfully. The agents' equilibrium behavior for the case of private information depends on their prior beliefs and the precision of their private information. When private information is accurate or prior beliefs high, optimistic agents invest immediately. This induces pessimistic agents to invest more aggressively to signal positive news to their opponent and encourage investment. Compared to the benchmark case, the pessimistic agent's signaling incentive increases his incentive to invest. When private information is very noisy and agents are initially not very optimistic, by contrast, the optimistic agents are hesitant and engage in a war of attrition, which delays their investment. Since only the optimistic types do so while pessimistic types stand by, the equilibrium gives rise to a positive feedback loop. The longer investment is delayed, the more agents become convinced that their opponent is a pessimist. In this way, information transmission and investments accelerate over time.

We thus conclude that private information can speed up the adoption of new, potentially risky, technologies in welfare-enhancing ways, while sharing of information can stifle adoption. This result has both normative and positive implications for a range of applications beyond the adoption of potentially risky new technologies. For instance, should countries exchange their prior information, based on which they evaluate the adoption of a risky technology, such as nuclear power generation, a new medical drug or vaccine? Is it in the lenders' interests to share their information about a given borrower? Our analysis suggests that the answers to these questions are far from clear-cut, as the welfare gains in terms of information aggregation stemming from the disclosure of private information may be outweighed by losses arising from exacerbated free-riding incentives.

1.1 Literature Review

We model experiential learning using the *strategic exponential bandit* framework (Keller et al., 2005). A key insight in this literature is that with publicly observable experimentation, learning tends to be inefficiently slow, as the agents have an incentive to delay investment to free-ride on the information provided by others. Keller and Rady (2015) considers a "bad-news" learning variant similar to ours, in which players face the possibility of a random breakdown. In contrast to our paper, however, players have no private information, and payoffs are publicly observable. Bonatti and Hörner (2017) studies a variant of this model with public payoffs and private actions. They show that players are more prone to deviating to shirking experimentation than with publicly-observed actions, because, with public actions, such a deviation makes players more pessimistic than they would be absent the deviation, and hence less inclined to experiment. Thus, privately observed actions lead to lower ex-ante expected welfare.[1] In our setting, in contrast, an informational asymmetry arises due to private signals at the outset, while actions are publicly observed. In this regard, our setup is more similar to Décamps and Mariotti (2004), which considers a strategic-experimentation model in which players are privately informed about their own costs. Players prefer their opponent to invest first, as this generates public information about the payoff-relevant state. They show that private information about costs exacerbates the free-riding problem, because players have an additional incentive to delay investments in order to convince each other that they face a high cost. The players' incentives thus work in the opposite direction from our paper, as we assume that private information relates to the payoff-relevant state, which gives players an incentive to invest earlier to signal optimism.[2]

A second strand of literature, following Bikhchandani et al. (1992b) and Banerjee (1992), studies aggregation of private information through "*social learning*" from the actions of others. A key insight is that observational learning from actions fails to aggregate private information because agents' private information may be outweighed by what they have observed. Thus, they have an incentive to ignore their own private information and to follow the "herd" when choosing their (observable) action, implying that their private information is never revealed. Chamley and Gale (1994) and Murto and Välimäki (2013) consider invest-

[1] Bonatti and Hörner (2011) analyze strategic experimentation with private actions in a *good-news* setting, and show that, in that setting, players are more prone to shirking when actions are observable. Thus, ex-ante expected welfare is higher when actions are not publicly observed in that setting.

[2] In Bobtcheff and Levy (2017), the firm also has an incentive to signal optimism to the capital market. The firm privately knows the arrival rate of private, fully conclusive, pre-investment bad news, and signals optimism by investing early.

ment models of social learning in which the time of an agent's observable and irreversible decision is endogenously determined in equilibrium. In both papers, information is inefficiently aggregated because investors have incentives to delay their respective exit decisions so as to acquire more information by observing the behavior of others.[3] In our paper, strategic delay occurs as well, but it is driven by the players' incentive to free-ride on each other's experimentation. We show that this delay is mitigated when players have an incentive to signal optimistic private information about the underlying state.

Following Rosenberg et al. (2007), several papers, such as Murto and Välimäki (2011) and Rosenberg et al. (2013), analyze *strategic experimentation with privately observed payoffs*. In these papers, the players' only choice is an irreversible stopping decision. Due to the irreversibility of exit, there are no free-riding incentives, and thus public information is unequivocally good for welfare in these settings. In contrast, we show that, when players' actions are reversible, private information among agents counteracts the free-riding incentives and can thus be welfare-enhancing. Heidhues et al. (2015) investigate the role of communication in a setting with private payoffs and reversible, publicly observable, actions, allowing for cheap talk between players. They show that experimenting agents can use communication as a tool to incentivize effort by threatening to withhold future information, thereby increasing total surplus relative to the public-information benchmark. In our paper, in contrast, actions and payoffs are publicly observed, and players motivate each other by signaling optimism.

More similar to our setting, Dong (2021) introduces a *privately informed player into the good-news exponential-bandit setting* of Keller et al. (2005). In particular, actions and outcomes are publicly observable, and one player observes a private signal about the payoff-relevant state at the outset. Consistently with our findings, Dong (2021) shows that the informed player has an incentive to experiment more aggressively so as to signal optimism. Yet, due to the informed player knowing all available information, there is no problem regarding information aggregation, and, as a result, the author finds that private information is better than public information when signals are very informative. In contrast, we show that when both players receive private signals, the failure of information aggregation can lead to large social losses when signals are very precise, and thus the encouragement that arises due to private information leads to welfare improvements if signals are not too informative.

A number of papers consider learning from others' actions and outcomes in models with additional application-specific *payoff externalities*. Moscarini and Squintani (2010) consid-

[3] Ex-ante welfare may be improved in these settings by introducing taxation to make early investments more informative (Heidhues and Melissas, 2012), or by promoting communication between players, Gossner and Melissas (2006).

ers a model of a winner-takes-all R&D competition in which firms observe an initial private signal and decide when to exit irreversibly. They show that the aggregate duration of experimentation is longer under private information. Since the players' only decision is the time of their irreversible exit, there are no signaling motives, in contrast to our setting. Bobtcheff et al. (2021) studies a winner-takes-all model with "bad-news" learning. They show that public information about payoffs is detrimental when competition between players is strong.

Cetemen (2021) analyzes a continuous-time, two-player, team-production problem with initial private information and a finite deadline. Specifically, the players' payoff flows depend on the product of the unknown state of the world and the sum of their efforts. At each instant, players choose among a continuum of actions; throughout, they observe a noisy signal of aggregate effort, while only observing payoffs at the end. Cetemen (2021) shows that signalling incentives counteract free-riding incentives, enabling an approximation of the first best if there is no discounting and the time horizon is sufficiently long, so that a setting with private information may be welfare-superior. In our paper, we show that, by virtue of a similar positive welfare effect of signalling, less initial information can be better even in a model of pure (positive) informational externalities. In Margaria (2020), agents' private signals arrive over time and represent fully conclusive bad news. Thomas (2020) studies strategic experimentation with privately observed payoffs when players compete for the use of a single shared safe arm, and finds that, for many priors, experimentation is more efficient than with public payoffs. Indeed, it is shown that signaling incentives counteract free-riding incentives, and that a setting with private information may as a result be welfare-superior. In our paper, we show that, by virtue of a similar positive welfare effect of signalling, less initial information can be better even in a model of pure (positive) informational externalities.

The potential welfare gains from private information are driven by signaling incentives and an effect that has been labeled the"smoothing effect of uncertainty"(Morris and Shin, 2002). Welfare gains through signaling incentives have been shown by Hermalin (1998) in a static team-production model with payoff externalities. Teoh (1997) demonstrates how uncertainty can improve welfare in a model of public-good provision, where the marginal return to agents' investments is determined by an uncertain state of the world. The author shows that non-disclosure of information may increase ex-ante welfare when the investment has marginally diminishing returns because the loss resulting from a reduction in investment after the release of bad news outweighs the benefits from increased investment when the information is favorable. This mechanism is also present in our paper: when bad news is publicly disclosed, free-riding increases, leading to an over-proportional reduction in the expected value of an investment.

2 Model

There are two agents, indexed $i = 1, 2$. Time $t \in \mathbb{R}_+$ is continuous, with an infinite horizon. Future payoffs are discounted at the common discount rate $r > 0$.

Risky investment Each agent decides when to invest an amount $I > 0$ to initiate a project with uncertain returns. The project generates a stochastic payoff stream that depends on an unknown state of the world $\theta \in \{G, B\}$, which is either "good" ($\theta = G$) or "bad" ($\theta = B$). While the project is operational, it yields a flow return of $y > rI$ in either state. However, when the state is bad, publicly observable accidents occur at random times corresponding to the jump times of a time-homogeneous Poisson process with parameter $\gamma > 0$. Accidents never occur in state G. Conditionally on the state being B, the arrival times of accidents are independent across agents. An agent whose project causes an accident incurs a lump-sum cost of $c > 0$. We assume that each agent can abandon and reinvest in the project at any time.

Prior information The prior probability that the state is good is given by p_0. Players observe some additional information in the form of binary signals $s_1, s_2 \in \{h, l\}$, which are i.i.d. conditionally on θ. The probability that the signal is h (high) in state G is equal to the probability that it is l (low) in state B; we denote this probability by $\rho \in (\frac{1}{2}, 1)$.

Structure of the investment game We model the continuous-time environment as a repeated stopping game with multiple "phases." At the beginning of the first phase, the agents decide how long to wait before making the investment, conditionally on the other agent not having invested yet. The initial stage ends after the first agent invests or both invest simultaneously. If only one agent invests, then the agent who invested is called the "leader," and the other the "follower." In the second phase, each agent who invested decides if and when to exit while an agent who did not invest decides when to enter, each conditionally on the other agent not moving first. Later phases proceed in a similar fashion. We assume that $\gamma c > y$, so that after an accident has arrived and players have learned that $\theta = B$, it is a dominant action for players not to invest, or, respectively, to exit a prior investment immediately. We take this as given in our subsequent analysis, and treat all histories following an accident as terminal histories.

Histories and Strategies We define an investment history at time $t \geq 0$ to be a profile $h_t = ((\mathcal{S}_1, \tau_1), \ldots, (\mathcal{S}_{n_t}, \tau_{n_t}))$ with $0 \leq \tau_1 \leq \ldots \leq \tau_{n_t} \leq t$, where τ_k for each $k = 1, \ldots, n_t$ represents a "switching time" at which each agent in $\mathcal{S}_k \subseteq \{1, 2\}$ has changed his investment decision, and $n_t \in \mathbb{N}$ represents the total number of instances of such changes in the past. We make the technical assumption that, at each instant, entry is observed before exit, so that when two agents switch at the same time in opposite directions, play proceeds to the phase corresponding to only the entering agent switching.

We refer to n_t as the number of *phases* at history h_t. A behavioral strategy for agent i is then given by a family of cumulative distribution functions $\{F_i(\cdot|s_i, h_t)\}_{h_t \in H_t}$ with $F_i(t'|s_i, h_t) = 0$ for all $t' < \tau_{n_t}$. Here, $F_i(t'|s_i, h_t)$ represents the probability that agent i with signal s_i takes action (invests or exits) before or at time $t' \in [\tau_{n_t}, \infty]$ following investment history h_t, conditionally on the other agent $-i$ not taking action before t'.

Payoffs A profile of behavioral strategies induces a distribution over switching times for each agent i. Denoting by $(\tau_k^i)_{k \in \mathbb{N}}$ the random investment and exit times for player i, the expected normalized payoff for agent i at any time t is

$$\mathbb{E}_t \left[\sum_{k=1}^{\infty} \left(\int_{\tau_{2k-1}^i \vee t}^{\tau_{2k}^i \vee t} e^{-r(\xi - t)} r(y - \mathbf{1}_{\{\theta = B\}} \gamma c) d\xi - \mathbf{1}_{\{\tau_{2k-1}^i > t\}} e^{-r(\tau_{2k-1}^i - t)} r I \right) \middle| s_i, h_t \right]. \quad (1)$$

We say that an agent is "invested" at any history at which he has performed an odd number of switches. Otherwise this agent is called "out." While agent i is "out," i.e., on time intervals $(0, \tau_1^i), (\tau_2^i, \tau_3^i), \cdots, (\tau_{2k}^i, \tau_{2k+1}^i)$, he accrues no payoff.[4]

Equilibrium concept The posterior belief of player $i = 1, 2$ is a joint probability distribution in $[0, 1]^2$ over the state θ and the other agent's signal. We focus on symmetric perfect Bayesian equilibrium. A *perfect Bayesian equilibrium* is a pair of behavioral strategies, together with a belief system for each agent, which assigns a probability distribution over signals and the state of the world at each history, such that (i) each agent's strategy maximizes his expected payoff, given his belief over the state and the other agent's signal and (ii) beliefs are updated via Bayes' rule at any history that lies in the support of the distribution over histories induced by the agents' strategies. We shall say that a perfect Bayesian equilibrium is *symmetric* if the players' equilibrium strategies prescribe the same (mixed) action whenever they have the same beliefs and are in the same *mode*, that is, they

[4]We assume that any history with an infinite number of switches gives both players a payoff of $-\infty$.

are either both invested or both out.[5]

Throughout, we assume that the investment cost I that is required to initiate a project is large enough to ensure that agents remain invested after they initiated their project (unless an accident occurs), even if the other agent deviates from his equilibrium strategy.

Notation Throughout, we denote by p_t the (history-dependent) public posterior belief that $\theta = G$ at time t, i.e., the belief held by a hypothetical outside observer, who started out with a prior belief of p_0 and observed the public history but did not know the realizations of the initial signals. By the same token, we denote by q_{it} the public posterior belief assigned to agent i's type being g (we omit the index i whenever the belief is the same for each agent). Furthermore, we write $p_t(s)$ and $q_{it}(s)$ for the respective posterior probabilities conditional on a single signal $s \in \{g, b\}$, and, analogously, $p_t(s, s')$ for the posterior probability about the state, conditional on a pair of signals $(s, s') \in \{g, b\}^2$. Note that, since signals are i.i.d. and symmetric, we have $p_t = p_t(g, b) = p_t(b, g)$.

3 Planner solution

We begin by considering the case of a planner who chooses investment times in order to maximize the sum of the agents' payoffs based on the realizations of both signals. The agents thus share a common belief at the outset, and we denote their common belief by $\check{p}_0 := p_0(s_1, s_2)$. The agents' shared belief about the realization of the state of the world is updated based on the observed actions and payoffs once the first agent makes an investment and triggers a payoff flow. In the absence of any accidents following an investment, the posterior belief $\check{p}_t$ continuously evolves following the differential equation

$$\frac{d\check{p}_t}{dt} = k_t \gamma \, \check{p}_t (1 - \check{p}_t),$$

where $k_t \in \{1, 2\}$ denotes the number of players being invested at instant t. The posterior belief thus gradually increases as long as no accident occurs; it is not surprising therefore that it is socially optimal for an agent to remain invested indefinitely until an accident occurs. Whether or not investing is socially optimal thus depends on the initial belief $\check{p}_0$. To state

[5] While our definition of symmetry may be reminiscent of Markov perfect equilibrium, it is in fact more permissive, in the sense that it allows players to choose different (mixed) actions at different histories even if they lead to the same belief and mode; symmetry only requires that they both choose the *same* mixed action at all histories at which they have the same beliefs and are in the same mode.

the following theorem, which characterizes the socially optimal investment strategies, we define the log-likelihood ratio $\phi(p) := \ln(p) - \ln(1-p)$.

Theorem 1 (Planner Solution). *There exist thresholds $p_1^* \in (0,1)$ and p_2^* with $p_1^* < p_2^* < 1$, so that it is socially optimal for both agents to invest immediately if $\check{p}_0 \geq p_2^*$ and never to invest if $\check{p}_0 < p_1^*$. If $p_1^* \leq \check{p}_0 < p_2^*$, it is socially optimal for one agent to invest immediately, and for the second agent to invest with delay $\tau^s(\check{p}_0) = (\phi(p_2^*) - \phi(\check{p}_0))/\gamma$.*

All proofs are found in the Appendix. The theorem shows that staggered investment is optimal for intermediate values of the interim beliefs due to investment costs. Since the initial investment costs required to start a project cannot be recovered after a failure, it is socially preferable to start only one project initially, which then generates a flow of information on which the start of the second project can be conditioned. In this way, staggered investment lowers the loss from making effectively irreversible investments in the bad state.

4 Equilibrium analysis

We now consider the setting in which the agents choose the timing of investments strategically and study how the distribution of information affects equilibrium investments. First, we analyze the case of public information, in which the signals are made public at the beginning of the game. Then, we consider the case of private information in which each agent observes a signal.

We start with a preliminary result. Consider a history at which one agent is invested ($k = 1$). The follower benefits from the leader's experimentation in this case, because of the possibility that the leader experiences an accident. If the leader experiences an accident, the follower learns that the state of the world is bad without incurring any losses. Depending on the follower's posterior belief about the state, it may thus be profitable for him to delay the investment. Assuming that the follower delays his own investment by some duration τ (at which he is sufficiently confident that the state is good), the expected net present value before paying investment costs I for the leader at any belief p is given by

$$v_l(p,\tau) = py + (1-p)\left((1-e^{-(r+\gamma)\tau})\lambda_1 + e^{-(r+\gamma)\tau}\lambda_2\right)(y-\gamma c) - rI, \tag{2}$$

where by $\lambda_k = r/(r + k\gamma)$ we denote the marginal value of a discounted unit payoff stream up to termination at a random time arriving at constant rate $k\gamma$ for $k = 1, 2$. By the same token, assuming that the leader remains invested indefinitely, the expected present value of the follower when delaying the investment by a duration τ is given by

$$v_f(p,\tau) = e^{-r\tau}py + e^{-(r+\gamma)\tau}(1-p)\lambda_2(y-c\gamma) - (p+(1-p)e^{-\gamma\tau})e^{-r\tau}rI. \tag{3}$$

The following lemma reports basic properties of the functions v_l and v_f.

Lemma 1. *The function $v_l(p,\tau)$ is linearly increasing in p, convex and decreasing in τ for every $p \in (0,1)$ and supermodular in (p,τ). The function $v_f(p,\tau)$ is linearly increasing in p and has a single peak in τ at*

$$\tau^*(p) = \begin{cases} \left(\phi(p_f^*) - \phi(p)\right)/\gamma & \text{if } p < p_f^* \\ 0 & \text{if } p \geq p_f^* \end{cases} \tag{4}$$

for every $p \in (0,1)$, where

$$p_f^* = \frac{\lambda_1(r+\gamma)I + \lambda_2(\gamma c - y)}{\lambda_1(y+\gamma I) + \lambda_2(\gamma c - y)}. \tag{5}$$

Moreover, we have $v_l(p,\tau) \leq v_f(p,\tau)$ which holds with equality if and only if $\tau = 0$.

In addition to characterizing the values of investment for the leader and the follower, respectively, the lemma provides a characterization of the follower's optimal delay $\tau^*(p)$, which is the time it takes for the belief to travel from p to p_f^*, given one agent remains invested and no breakdown occurs. The fact that the follower value is greater than the leader value at any belief $p < p_f^*$ shows that information spill-overs generate a second-mover advantage.

We write $v_f^*(p) = v_f(p,\tau^*(p))$ and $v_l^*(p) = v_l(p,\tau^*(p))$ for the values of the leader and the follower, respectively, given the follower uses the optimal delay. Since τ^* is weakly decreasing in p, and v_l and v_f are strictly increasing in p as well as decreasing in τ, it follows that v_l^* and v_f^* are strictly increasing functions in p. Moreover, v_l^* is continuous, positive if $p = 1$, and negative if $p = 0$. Hence, it has a unique root on $(0,1)$, which we denote by p_l^*. Note that, by definition of p_f^* and p_l^*, we have $v_f^*(p_f^*) = v_l^*(p_f^*) > 0$ and thus $p_l^* < p_f^*$.[6] One furthermore shows that $p_1^* < p_l^*$ and $p_2^* < p_f^*$.

[6] That $p_l^* < p_f^*$ is somewhat related to the effect in Melissas (2005), where slower information production later boosts current information production.

4.1 Public Information

We now study the case of full disclosure in which both agents observe the realization of both signals, focussing on symmetric equilibria. As indicated above, there is a second-mover advantage to investing due to information spill-overs: each agent prefers to learn from the experience of the other and to avoid losing money on a failing investment. This second-mover advantage provides each agent with an incentive to strategically delay his investment in the hope that the other agent invests first.

For an intermediate range of beliefs, this results in the agents choosing the timing of their investment at random in equilibrium, namely at a constant rate that renders the other agent just indifferent between investing immediately and delaying the investment by an instant. In order to state the theorem, which, for high investment costs I, characterizes the unique symmetric equilibrium for symmetrically informed agents, we define

$$\beta^*(p) = \max\left\{\frac{rv_l^*(p)}{v_f^*(p) - v_l^*(p)}, 0\right\}. \tag{6}$$

The equilibrium is then given as follows.

Theorem 2 (Public information)**.** *There exists $I^* \in (0, y/r)$ such that, for all $I > I^*$, there is a unique symmetric equilibrium, in which neither agent exits before the arrival of an accident. In this equilibrium, for all times $t \in [0, \infty)$, on path as well as off path:*

1. *If $\check{p}_t \geq p_f^*$, both agents invest immediately.*
2. *If $p_l^* < \check{p}_t < p_f^*$, each agent invests at constant rate $\beta^*(\check{p}_0)$ given by Equation (6) in the first phase, while the follower starts the project with delay $\tau^*(\check{p}_0)$.*
3. *If $\check{p}_t \leq p_l^*$, neither agent invests.*

The basic equilibrium structure mirrors that of the symmetric equilibrium in Keller and Rady (2015), in the sense that there are two belief thresholds with the property that there is no experimentation below the first, and maximum experimentation above the second threshold, with randomization for all beliefs that lie between these thresholds. While the equilibrium shares the property of staggered investments with the planner's solution, there is, in contrast to the planner's solution, a period of inefficient delay prior to the initial investment for beliefs $\check{p}_0 \in (p_l^*, p_f^*)$. Moreover, the equilibrium exhibits too little experimentation relative to the efficient benchmark. On the one hand, since $p_1^* < p_l^*$, there are values of the interim belief at which experimentation is socially valuable but does not arise in equilibrium. Second, since $p_2^* < p_f^*$, delay of the second investment is inefficiently long. Inefficiencies arise in equilibrium due to free-riding incentives: agents benefit from the information generated by their competitor's experimentation, failing fully to internalize the social value of their own experimentation. The incentive to free-ride thus leads to inefficiently long delays in investment by the follower and sluggish initial investment, as each agent prefers the other one to invest first. For intermediate prior beliefs, the first phase of the game is strategically related to "war-of-attrition" games (see, e.g., Bulow and Klemperer, 1999), which are timing games in which the players incur a cost until their time of exit, and reap a reward if they endure the cost longer than their opponent.

4.2 Private Information

We now turn to equilibria in the case of private information, in which each agent privately observes a single signal. The equilibria differ from the case of publicly observed signals in that the presence of private information exacerbates uncertainty and introduces signaling incentives.

In the equilibria we characterize, the way private information is revealed depends crucially on whether the most optimistic interim belief $p_0(g, g)$ exceeds the follower threshold p_f^* or not. If $p_0(g, g) \geq p_f^*$, optimistic agents invest without delay, so that all private information that is revealed in equilibrium is revealed in a lump at time zero. Otherwise, optimistic agents delay their initial investment while pessimistic agents wait, so that private information is aggregated continuously over time.

The following result characterizes three different classes of equilibria and conditions for their existence that depend on the prior belief $p_0 \in (0, 1)$ and the signal precision $\rho \in (0.5, 1)$. While we restrict ourselves to high investment costs I, the theorem covers all combinations of the parameters (p_0, ρ). Note that, depending on the parameters, the rate $\beta^*(p_0(b, b))$ may be 0.

Theorem 3 (Symmetric equilibrium with private information)**.** *There is a $I^{**} \in (0, y/r)$ such that, for all $I > I^{**}$, there exists an equilibrium in which no agent exits after having been invested for a positive amount of time prior to an accident, followers' delay is given by the function τ^* applied to their belief about θ, and the following holds.*

(1.) *Suppose $p_0 \geq p_l^*$ and $\rho \in (0.5, 1)$ are such that $p_0(g, g) \geq p_f^*$. Then, there exists a symmetric equilibrium in which type g invests immediately. Type b of each agent invests* *immediately with some probability* $\eta^* \in [0,1]$, *while with probability* $1-\eta^*$, *he invests at a random time arriving at constant rate* $\beta^*(p_0(b,b))$.

(2.) *Suppose* $p_0 < p_l^*$ *and* $\rho \in (0.5,1)$ *are such that* $p_0(g,g) \geq p_f^*$. *Then, there exists a symmetric equilibrium in which type* g *invests immediately with some probability* $\nu^* \in [0,1)$, *while with probability* $1-\nu^*$, *he does not invest. Type* b *never invests.*

(3.) *Suppose* $p_0 > 0$ *and* $\rho \in (0.5,1)$ *are such that* $p_0(g,g) < p_f^*$. *Then, there exists a symmetric equilibrium in which type* g *of each agent invests at rate* $\mu_t^* \geq 0$ *given by* (23). *Type* b *delays investment until (possibly infinite)* $t^* > 0$, *given by* (24), *and invests at constant rate* $\beta^*(p_0(b,b))$ *thereafter.*

We refer to the first two types of equilibria as equilibria with “immediate investment,” and to the latter as an equilibrium with “delayed investment.” Whether the equilibrium exhibits delayed investment depends on the relative location of $p_0(g,g)$ vs. p_f^*.

Equilibria with immediate investment arise either when signals are very informative or if the prior belief is very high. Such equilibria may be pooling, partially separating, or fully separating. A high prior belief and weak signals result in pooling, where each agent invests immediately.

Indeed, if the interim belief of pessimistic agents, conditional on their own signal, is high enough, then it is always optimal for them to invest immediately. On the other hand, when signals are highly informative, the equilibrium tends to be partially or fully separating. Intuitively, an informative good signal provides a strong incentive for an agent to invest, while an informative bad signal makes investing costly. However, immediate investment communicates good news that makes one’s opponent more willing to invest, which, in turn, generates a positive informational externality. As a result, pessimistic agents have incentives to pretend to be optimistic, in order to encourage the other agent to experiment.

Equilibria exhibit delayed investment when the prior is not too high and signals not too informative. In an equilibrium with delayed investment, optimistic agents engage in an attrition game, while pessimistic agents simply wait; indeed, as in the case of public information, optimistic agents delay investment because they benefit from the possibility that their opponent invests first and subsequently provides free information.

In contrast to the case of public information, however, in equilibrium, an optimistic agent invests at a rate that keeps his opponent’s *optimistic type* indifferent, irrespectively of his opponent’s true type, while, with public information, agents keep each other’s true types indifferent. Moreover, because only optimistic agents invest with positive probability, the

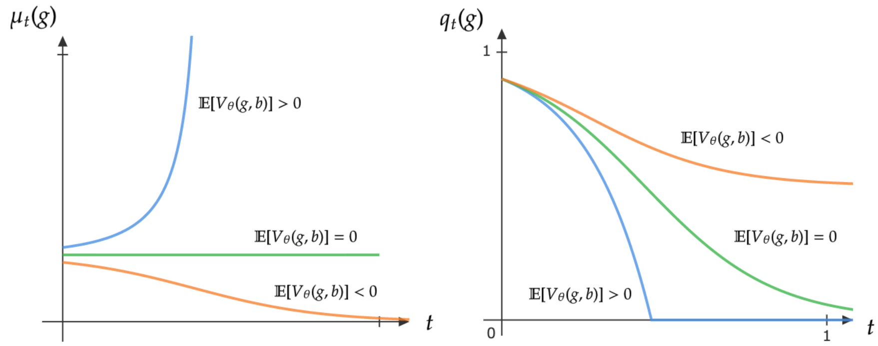

Figure 1: Three branches of equilibrium investment rates (left panel) and the corresponding evolution of the belief (right panel).

agents learn about one another's types while they wait. As long as neither agent invests, each agent becomes gradually more certain that the other one has observed a bad signal. This gradual change in beliefs about the other's type, in turn, affects the agents' incentives to invest. The interaction between belief updating, incentives and actions thus creates a learning feedback loop that either accelerates or dampens the speed of learning.

The effects of the feedback loop can be seen in the dynamics of the investment rates illustrated in Figure 1. Here $\mathbb{E}[V_\theta(s_i, s_{-i})]$ denotes the expected equilibrium payoff from being a leader in state θ for an agent with signal s_i, when his opponent's signal is s_{-i}. The upper branch in the left panel of the figure corresponds to the case in which the expected equilibrium payoff $\mathbb{E}[V_\theta(g, b)] > 0$ from being an optimistic leader, conditional on the opponent's signal being bad —i.e., the worst possible payoff for type g— is positive. Note that this expected payoff is independent of time, since the signal pair (s_i, s_{-i}) encapsulates all information that is exchanged in the first phase, so that $p_t(s_i, s_{-i}) = p_0(s_i, s_{-i})$. In this case, an agent of type g wants to invest regardless of his opponent's type. In equilibrium, each optimistic agent invests at a rate that makes his optimistic opponent just indifferent between investing and waiting. The longer an optimistic agent waits for the other to invest first, the more convinced he becomes that the other's delay is due to his signal being bad. In equilibrium, therefore, optimistic agents must increase their rates of investment in order to continue to make the good type of the other agent indifferent.

This increase, in turn, accelerates the decline in beliefs, as shown in the lower branch of the right panel of Figure 1, which requires a further increase in the investment rate of optimistic agents. The result is an escalating feedback-loop between investment and learning rates, which causes investment rates to shoot off to infinity by some finite time t^*, so that all private information is revealed by t^*.

The lower branch in the left panel of Figure 1 corresponds to the case in which type g's worst-case continuation payoff $\mathbb{E}[V_\theta(g,b)]$ from an immediate investment is negative. As before, the longer an optimistic agent waits for the other to invest first, the more convinced he becomes that the other's deferral is due to his signal being bad.

In order for optimistic agents to continue to be indifferent between waiting and investing, they must now decrease their rate of investment gradually. This reduction again triggers a feed-back loop, in which decreasing investment rates slow down learning, as shown in the upper branch of the right panel of Figure 1, which in turn dampens investments and so on. Investment rates eventually tend to 0 and the agents' private information is never fully revealed.

If $\mathbb{E}[V_\theta(g,b)] = 0$, finally, a type-$g$ agent would be indifferent between investing and staying out if he knew his partner to be of type b. In this case, agents of type g invest at a constant rate in equilibrium, so that agents' beliefs that their partner is of type g decline over time, yet all private information is only revealed in the limit as $t \to \infty$ as shown in the middle branch of the right panel of Figure 1.

4.3 Welfare comparison: public vs. private information

In comparing the equilibrium outcomes with and without information asymmetries, it is natural to ask which environment is more desirable from an efficiency standpoint. Naïve logic may suggest that more transparency should unambiguously lead to better outcomes, as it allows the agents to make better-informed decisions. The *World Medical Association's* "Ethical Principles for Medical Research Involving Human Subjects," for instance, state that "Researchers have a duty to make publicly available the results of their research on human subjects."[7] While there may be many good reasons for such a policy, which are not captured by our model,[8] they may conceivably be to some extent counter-balanced by the aforementioned positive side-effects of private information (though these countervailing effects are unlikely to be of first-order importance in the case of medical-drug trials).

[7] Available at https://www.wma.net/policies-post/wma-declaration-of-helsinki-ethical-principles-for-medical-research-involving-human-subjects/ (accessed on June 25, 2020). We are indebted to Kaustav Das for pointing this out to us.

[8] E.g., if pharmaceutical firms were allowed only to disclose favourable test results, these would be very hard to interpret if one did not know how many unfavourable test results they may have received.

We find that private information is socially preferable if investment costs are substantial, the prior belief that the state is good is high enough, and the signals are not too informative. Indeed, denote by $W_0(s^1, s^2)$ the expected social surplus generated in the unique symmetric equilibrium under public information, when $p_0(s_1, s_2)$ is the common initial belief that the state is good. Let $\tilde{W}$ denote the ex-ante expected social surplus generated in a symmetric equilibrium in which each agent's signal is private information. We then have the following

Theorem 4. *Fix $p_0 > p_l^*$. There exists $\rho^* > 1/2$ and $\bar{I} \in (0, \frac{y}{r})$ such that, for all signal precisions $\rho \in (1/2, \rho^*)$ and investment costs $I \in (\bar{I}, \frac{y}{r})$, we have $\tilde{W} \geq \mathbb{E}[W_0(s_1, s_2)]$. For $p_l^* < p_0 < p_f^*$, the inequality is strict.*

Theorem 4 states that private information is more desirable from an efficiency perspective if (1) the prior information is sufficiently optimistic, and (2) signals are not too informative. Indeed, for these parameters, there is under-investment in expectation under symmetric information, and the informational asymmetry leads to increased expected investment rates. If signals are very informative, by contrast, free-riding is not a big problem under symmetric information, which may perform better in this case. Indeed, if $p_0 \geq p_f^*$, for instance, the equilibrium in Part (1.) of Theorem 3 amounts to a pooling equilibrium in which players always invest right away. Yet, if ρ is high enough that $p_0(b, b) < p_1^*$, this amounts to inefficient over-investment, as players ought to refrain from investing if they both receive a bad signal, an outcome that is achieved under symmetric information only. By the same token, if the prior belief $p_0 < p_l^*$, asymmetric information will lead to less investment than symmetric information—this will be the case, for instance, in the equilibrium of Part (2.) of Theorem 3.

The condition on the parameters ensures that it is socially optimal for both agents to invest (though it may be socially optimal for the second investment to occur with delay). If $p_0 \geq p_f^*$, there always exists a range of signal precisions guaranteeing that both types of agent invest immediately under private information, as in the efficient benchmark. If $p_l^* < p_0 < p_f^*$, then there are signal precisions such that the symmetric equilibrium under public information has delayed entry. In this case, a mixed-strategy equilibrium arises both under public and private information. In this equilibrium players invest at a rate that keeps the actual type of their opponent indifferent, if information is public. If information is private, by contrast, type g of player i invests at a rate that would keep type g of player $-i$ indifferent, even if player $-i$ happens to be of type b. The proof establishes that, for certain parameters, the welfare benefit of this increased investment rate of the g-types will overwhelm the welfare loss both from the reduced investment rate of b-types and from players' less precise knowledge of the underlying state.

Finally, we should stress that Theorem 4 is indeed a novel result. It is a well-known fact that more information can lead to socially inferior equilibrium outcomes in games with payoff externalities, as uncertainty will typically relax incentive-compatibility constraints. In our model, however, one agent's actions affect the other's belief but not his payoff, so that the basic logic behind the widely familiar result does not apply.

4.4 Applications

4.4.1 Technology adoption

Our model of strategic investments can offer insights into the role of observational learning in technology diffusion. An extensive body of empirical research has found contagion effects and clustering of technology adoption decisions that are well explained by social learning. There is evidence to this effect for a variety of markets, including workers' saving decisions (Duflo and Saez, 2002; Ouimet and Tate, 2020), adoption of medical innovations by physicians (Burke et al., 2007; Manchanda et al., 2008; Iyengar et al., 2011), educational decisions (Bobonis and Finan, 2009; Carrell and Hoekstra, 2010), information about job opportunities (Beaman and Magruder, 2012), health (Kremer and Miguel, 2007; Oster and Thornton, 2011), agricultural technologies (Foster and Rosenzweig, 1995; Conley and Udry, 2010), consumer goods (Goolsbee et al., 2002), and policy making (Shipan and Volden, 2008; Gilardi, 2010).

A number of papers have looked specifically at the effects of learning spillovers on the rate of technology adoption. Foster and Rosenzweig (1995) study the decisions of Indian farmers to adopt high-yield crop varieties, and find that farmers experiment less when they are able to observe neighbors who produce more information on average. Munshi (2004) compares the adoption of high-yield variants between wheat and rice farmers in India, and argues that rice farmers tend to experiment more with new crop variants because their growing conditions are more idiosyncratic than wheat farmers', so that they have fewer opportunities to free-ride on their neighbors' experience. Conley and Udry (2010) go further by identifying the specific social links and communication patterns of pineapple farmers in Ghana in order to relate adoption decisions to the experiences of other farmers in their network. In their study, they find evidence that farmers adjust their inputs to align with those colleagues in their social network who have been surprisingly successful with a novel type of crop.

These papers view social learning purely as learning from others' experiences, as opposed to their beliefs or actions. There is strong evidence, however, that decision-makers are also heavily influenced by others' attitudes and choices. Anderson and Holt (1997) conduct a laboratory experiment, and find that, consistent with the theoretical prediction of Bikhchandani et al. (1992a), test subjects may follow the crowd rather than their prior information, thus taking others' actions into account when making their respective decisions. This economic model of learning from actions has been used to explain the empirical finding that adoption decisions are strongly influenced by "opinion leaders," i.e., community members with a superior status, knowledge or experience. Early work by Coleman et al. (1957) finds that early adoption of a new pharmaceutical drug by respected doctors increases the rate of adoption by other physicians. More recently, Miller and Mobarak (2015) conduct a field experiment on the adoption of non-traditional chimney stoves in Bangladesh and show that villagers tend to adopt more when opinion leaders unanimously adopt stoves and less when opinion leaders reject them. Similarly, Maertens (2017) finds that farmers are influenced in their adoption of new crops by the choices of early-adopting influential neighbors in their village.

There is also evidence that opinion leaders are aware of their role and internalize the effect of their own decisions on the choices of their peers. Coleman et al. (1957) observes that doctors with larger social networks tend to adopt new drugs earlier than those with smaller networks. This is consistent with our theory that doctors with large networks have a larger incentive to experiment with a risky technology, in order to promote their adoption by peers (in our story, because they value the information their peers produce when adopting the technology). Iyengar et al. (2011) similarly finds that physicians who are more influential according to sociometric measures tend to adopt a drug earlier than their less influential peers. The empirical evidence thus suggests that opinion leaders' decision whether to experiment with a new technology is a strategic choice that trades off the cost of experimentation with the personal loss from discouraging experimentation of others. Our results indicate that, unless there is strong evidence for the quality of a new technology, facilitation of information exchange among individuals may not increase the efficiency of information aggregation. Indeed, we show that private information may give rise to signalling incentives, which leads to more aggressive experimentation that counteract free-riding incentives, but at the cost of private information being imperfectly aggregated, which in turn can lead to socially excessive investment.

4.4.2 Teams and collaborations

We can also interpret our model as one of the formation of collaborations. Indeed, suppose the agents have the opportunity to engage in a joint project, but that it is uncertain whether it will succeed in the long run. To set up the project, the agents pay a contribution, and while the project is operational, they enjoy a continuous benefit from their participation. However, if the project fails, they then share the cost of remediation. Such an interpretation applies, for example, to international cooperative projects such as the European Organization for Nuclear Research (CERN) or the international space station ISS.

In this perspective, our model has implications for the comparison of group structures when the adoption of potentially risky technologies is at issue. Indeed, suppose our players could communicate at the beginning of the game. If signals were verifiable information, as would be the case if the players' backgrounds and expertise were similar, an optimistic player would always reveal his signal—the situation would therefore amount to our public-information case. By contrast, agents of vastly different background and expertise would not be able independently to evaluate each other's signals; communication would only be possible via cheap talk. As players always want their opponent to think that they are optimistic, there can be no informative cheap talk, and the situation amounts to our private-information case.

Our analysis may thus have implications for the composition of groups working on uncertain projects. Indeed, our results suggest that, for very uncertain projects, heterogeneous groups will be preferable. For instance, Abbasi and Jaafari (2013) show that research collaborations across different institutions are more strongly positively correlated with average yearly citations than internal collaborations. Barjak and Robinson (2008) show that the most successful teams in the life sciences are moderately engaged in international collaborations. Looking at publications in science and engineering by US-based researchers, Freeman and Huang (2015) show that greater ethnic and locational diversity of authors leads to better impact factors and citations.[9]

By the same token, while a lot of attention is focused on ways to improve the flow of information across countries (e.g., United Nations Conference on Trade and Development, 2014; Liverani et al., 2018), our analysis shows that this need not always be desirable. Indeed, while naïve intuition would intimate that information exchange between countries was always

[9]It is well known in the psychology literature (see, e.g., Bond et al. (1990)) that people detect lies more readily within their own culture than across cultures. If lies can be detected, signals amount to verifiable information, and we are in our public-information setting; otherwise, communication is tantamount to cheap talk, and we are in our private-signals setting.

beneficial for welfare, our analysis suggests that the effect can cut both ways. This is because information exchange will remove the encouragement effects of private information that we identify, and may thus have adverse effects on welfare, as Theorem 4 shows: as free-riding effects are exacerbated as a result of information sharing, inefficient delays in the adoption of the technology may worsen. We thus find that it is not a forgone conclusion that transparency is desirable in any and all contexts; to decide whether it is requires additional arguments arising from the particular application at hand. Indeed, our analysis suggests that private information (heterogeneous or *rotating* teams) is called for when signals are none too precise, in particular, when it is *not* the case that *two* signals may suggest a radically different course of action from the prior information. If the latter is the case, it is better to guarantee that the players have access to the information stemming from *both* signals. Thus, we should expect heterogeneous teams to be particularly advantageous for endeavours in which progress tends to be incremental, as would be the case in most sciences.

5 Conclusion

We propose a tractable model of strategic experimentation with private information and bad-news learning in the presence of non-negligible switching costs. We derive the unique symmetric equilibrium in the case of symmetric information. We proceed to construct symmetric equilibria for the case of privately observed signals, which exhibit either immediate or randomly delayed investment.

Finally, we show that the welfare gain from signaling can overwhelm the welfare loss from less precise information, so that equilibrium surplus can be higher under private information. This suggests that some initial secrecy may be beneficial in countering under-experimentation with a new technology that may be prone to breakdowns.

There are a number of natural extensions we do not address in this paper. For example, we have assumed signals to be symmetric in the sense that the probability that a signal is correct ρ does not depend on the state of the world θ. While we conjecture that our main qualitative results would continue to hold if we relaxed this assumption and ρ were a function of θ, the simplifications arising from $p_t = p_t(g, b) = p_t(b, g)$ would of course be lost. In particular, we should expect additional case distinctions to arise, which will depend on whether $p_0(g, b) = p_0(b, g) > p_0$ or $p_0(g, b) = p_0(b, g) < p_0$.

We also do not address (ex-ante) asymmetries between agents or situations with more than two agents. For public information, we conjecture that an equilibrium with many agents would be characterized by an increasing sequence of belief thresholds, where each indicates the belief at which the next investment takes place with certainty. Equilibria with private information will also be affected by signalling incentives and strategic uncertainty in this case, and we expect that many of the aspects described in this paper will carry over to a game with more than two players. However, the welfare implications are ambiguous, because the information-aggregation problem becomes more severe, while the social value of experimentation increases.

We have also ruled out side payments between the players. Allowing for transferable utility, and analyzing optimal tax or subsidy policies for early or late adopters of a potentially risky new technology, would be further interesting questions for future research.

6 Proofs

Proof of Theorem 1. It is obvious that, prior to an accident, it can never be optimal for either agent to switch more than once. Moreover, any delay up to the first investment cannot be optimal. Define $w(p,\tau)$ as the planner's value in the case in which one player invests immediately at time 0, and the other invests with delay $\tau \geq 0$:

$$w(p,\tau) = (1+e^{-r\tau})py + (1-p)\left[\lambda_1 + (2\lambda_2-\lambda_1)e^{-(r+\gamma)\tau}\right](y-\gamma c) \\ - (1+(p+(1-p)e^{-(r+\gamma)\tau}))rI.$$

We show that there exist thresholds $p_1^* \in (0,1)$ and $p_2^* \in (p_1^*,1)$ such that it is optimal for exactly one player to invest immediately when $p \in (p_1^*, p_2^*)$, and for both players to invest immediately if $p \geq p_2^*$. Suppose first that $p > p_1^*$. It follows from the definitions of λ_1 and λ_2 that $2\lambda_2 - \lambda_1 = \lambda_1\lambda_2$. Therefore, the marginal value of delaying the second investment is

$$\frac{\partial w(p,\tau)}{\partial \tau} = re^{-r\tau}[-p(y-rI) + (1-p)(\lambda_2(\gamma c - y) + (r+\gamma)I)e^{-\gamma\tau}]. \tag{7}$$

The expression in brackets is strictly decreasing in τ. This implies that $\partial w(p,\tau)/\partial\tau < 0$ for all $\tau \geq 0$ whenever $p > p_2^*$, where

$$p_2^* = \frac{(r+\gamma)I + \lambda_2(\gamma c - y)}{y + \gamma I + \lambda_2(\gamma c - y)}, \tag{8}$$

in which case it is socially optimal to make the second investment immediately. If $p \leq p_2^*$, then the socially optimal delay solves the first-order condition $dw(p,\tau)/d\tau = 0$. Thus, the optimal delay is given by

$$\tau^s(p) = \begin{cases} (\phi(p_2^*) - \phi(p))/\gamma & \text{if } p < p_1^*, \\ 0 & \text{if } p \geq p_1^*. \end{cases}$$

Define $w^s(p) = w(p, \tau^s(p))$. It is easy to check that $w^s(p)$ is a strictly increasing function with $w^s(0) < 0$ and $w^s(1) > 0$. Hence, $w^s(\cdot)$ has a unique root $p_1^* > 0$ and $w^s(p) > 0$ for $p > p_1^*$.

□

Proof of Lemma 1*.* (i) That v_l is linear in p is obvious from its definition in Equation (2) and v_l is increasing in p because it follows from $\gamma c > y > 0$ that the second term in Equation (2) is negative. To see that v_l is decreasing in τ, note that $\lambda_2 < \lambda_1$, and hence

$$\frac{d}{d\tau} v_l(p,\tau) = -(r+\gamma)(1-p)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) < 0$$

for all $p \in (0,1)$ and $\tau \geq 0$. That v_l is convex in τ for all $p \in (0,1)$ follows from

$$\frac{d^2}{d^2\tau} v_l(p,\tau) = (r+\gamma)^2(1-p)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) > 0.$$

Finally, supermodularity holds because

$$\frac{d^2}{dp d\tau} v_l(p,\tau) = (r+\gamma)(\lambda_1 - \lambda_2)e^{-(r+\gamma)\tau}(\gamma c - y) > 0.$$

(ii) Linearity of v_f in p is obvious from its definition in (3) and it is increasing in p because $\gamma c > y > 0$ implies that the first term in Equation (3) is positive and the second term is negative. For fixed $p \in (0,1)$, the derivative of v_f with respect to τ is

$$\frac{d}{d\tau} v_f(p,\tau) = -e^{-r\tau}\left[rpy - (r+\gamma)e^{-\gamma\tau}(1-p)\lambda_2(c\gamma - y)\right] + e^{-r\tau} rI[rp + (r+\gamma)(1-p)e^{-\gamma\tau}].$$

Let $\hat{\tau}(p)$ be the (finite) solution to the first order condition $dv_f(p,\tau)/d\tau = 0$. The second term in brackets is positive, so that $dv_f(p,\tau)/d\tau > 0$ if $\tau < \hat{\tau}(p)$ and $dv_f(p,\tau)/d\tau < 0$ if

$\tau > \hat{\tau}(p)$. Hence, v_f attains a global maximum at $\hat{\tau}(p)$. If $\hat{\tau}(p) \geq 0$, then solving

$$rp(y - rI) + (r + \gamma)e^{-\gamma\hat{\tau}^*(p_0)}(1 - p)(\lambda_2(y - c\gamma) - rI) = 0$$

for $\hat{\tau}(p)$ shows that $\hat{\tau}(p) = \tau^*(p)$. If $\hat{\tau}(p) < 0$, then $v_f(p, \cdot)$ is strictly decreasing on $[0, \infty)$, and therefore assumes its maximum at 0.

(iii) That $p_f^* > p_2^*$ follows by comparison of (5) and (8). The inequality $p_1^* < p_l^*$ follows from the fact that $w^s(p) = v_l(p, \tau^s(p)) + v_f(p, \tau^s(p)) > v_l(p, \tau^s(p)) \geq v_l(p, \tau^*(p))$, where the last inequality follows from $\tau^s(p) \leq \tau^*(p)$; this holds since $p_2^* < p_f^*$. Note that $\tau^s(p) > \tau^*(p)$ if $p < p_f^*$. Since $v_l^*(p_l^*) = 0$ and since $w^s(p)$ is an increasing function, we have $p_1^* < p_l^*$. □

Before we proceed to the proof of Theorem 2, we state and prove two lemmata. First, note that a necessary condition for simultaneous investment to be part of an equilibrium is that the payoff for each agent be non-negative. An agent's payoff from jointly investing immediately, given a posterior belief $\check{p}_t$, is given by

$$v_l(\check{p}_t, 0) = \check{p}_t y + (1 - \check{p}_t)\lambda_2(y - \gamma c) - rI. \tag{9}$$

This payoff is non-negative if and only if $\check{p}_t \geq \underline{p}$, where

$$\underline{p} = \frac{rI + \lambda_2(\gamma c - y)}{y + \lambda_2(\gamma c - y)}. \tag{10}$$

Clearly, there can be no initial investment in equilibrium when the prior belief $\check{p}_0$ lies below $\underline{p}$, since then the payoff from investing is necessarily negative for each agent.

Now, define $\hat{p}_l^*$ to be the lowest posterior belief at which the payoff of already being invested as the leader is non-negative, i.e. $v_l^*(\hat{p}_l^*) + rI = 0$. The following lemma shows that, for I sufficiently large, the thresholds $\hat{p}_l^*$, $\underline{p}$, p_f^* and p_l^* defined above satisfy the following chain of inequalities.

Lemma 2. *There is an $I_0 \in (0, y/r)$ such that, for $I \geq I_0$, we have*

$$\hat{p}_l^* < \underline{p} < p_l^* < p_f^* < 1.$$

Proof of Lemma 2. Note that for all $p < p_f^*$ we have $v_l^*(p) < v_l(p, 0) = v_f(p, 0) < v_f^*(p)$ with equality everywhere when $p = p_f^*$. By definition, we have $v_l(\underline{p}, 0) = 0$, $v_l^*(p_l^*) = 0$. Since v_l, v_f, v_l^*, and v_f^* are all continuously increasing functions, the first inequality $v_l^*(p) < v_l(p, 0)$ implies that $\underline{p} < p_l^*$. By definition of p_f^*, we have $v_f^*(p_f^*) > 0$, and thus the identity $v_l^*(p_f^*) =$

$v_f^*(p_f^*)$ implies that $p_l^* < p_f^*$. Finally, when $I \to y/r$ then $\underline{p} \to 1$, while $\hat{p}_l^* < 1$ is bounded away from 1. Hence, there is an $I_0 < y/r$, such that $\hat{p}_l^* < \underline{p}$. □

The lemma implies in particular that, for I sufficiently large, joint investment can never arise in equilibrium at any posterior belief below the threshold p_f^*, because each agent correctly anticipates that the opponent would never again exit following the investment (except after a failure). If agent 1, for example, was to invest at some posterior $\check{p}_t < p_f^*$, then agent 2 would prefer to wait and become the follower, knowing that agent 1 would not want to exit.

Lemma 3. *In every equilibrium with public signals, each agent invests immediately at any belief $\check{p} \geq p_f^*$.*

Proof. It is clear that for $\check{p} = 1$, it is a unique best-response for each agent to invest immediately. Because of switching costs, there also exists a threshold $p_0^\dagger \in (p_f^*, 1)$ close to one, such that an agent who is invested at a belief will not exit at any $\check{p} \geq p_0^\dagger$. At a history with posterior belief $\check{p} \geq p_0^\dagger$ at which exactly one agent is invested, the agent who is out will thus invest immediately. At a history with posterior belief $\check{p} \geq p_0^\dagger$ at which both agents are out, they anticipate that the other agent will invest immediately following their own investment, and thus each strictly prefers to invest immediately. Thus, in any equilibrium, both agents are invested at any belief $\check{p} \geq p_0^\dagger$. Because of switching costs, there exists an $\epsilon \in (0, p_0^\dagger - p_f^*)$ such that any history with posterior belief $\check{p} \geq p_1^\dagger := p_0^\dagger - \epsilon$ at which exactly one agent is invested, this agent would not exit. Again, by definition of p_f^*, the agent who is out would thus invest immediately. Again, if both agents are out, they would then invest immediately. Thus, both agents would invest immediately at any belief $\check{p} \geq p_1^\dagger$. The same argument applies for any threshold above p_f^* so that, in any equilibrium, both agents invest immediately at any $\check{p} \geq p_f^*$. □

Proof of Theorem 2. (1.) *Existence*: (i) Let $\check{p}_t \geq p_f^*$. The claim immediately follows from the definition of p_f^*.

(ii) Let $p_l^* \leq \check{p}_t < p_f^*$. If an agent who is invested exits, he receives the payoff $v_l^*(\check{p}_t)$ by construction. By the same argument as in Part (2.) below, it is never optimal for a leader to exit when $\check{p}_t > p_l^*$. Before either agent has invested, we have

$$v_f^*(\check{p}_t) > v_l^*(\check{p}_t) > 0,$$

which implies that each agent strictly prefers being the follower over being the leader, and each prefers being the leader over an outcome in which neither agent ever invests. By symmetry, each agent has to choose the same distribution over switching times. By standard arguments, the equilibrium distribution cannot have any atoms or gaps in its support. Thus, the investment rate $\beta^*(\check{p}_t)$ from Equation (6) characterizes the distribution that makes each agent indifferent between investing and not investing, which establishes the claim.

(iii) For $\check{p}_t \leq p_l^*$, the claim follows immediately from the definition of p_l^*.

(2.) *Uniqueness*: Consider a history with posterior belief $\check{p}_t \in [\hat{p}_l^*, p_f^*)$, at which exactly one agent, say agent 1, is invested. By Lemma 3, in any equilibrium, agent 2 invests immediately at any $s \geq t$ at which $\check{p}_s \geq p_f^*$. Thus, if agent 1 stays invested indefinitely, (or until an accident occurs,) the largest delay compatible with equilibrium is $\tau^*(\check{p}_t)$. The payoff for agent 1 is therefore no less than $v_l^*(\check{p}_s) + rI$ at each $s \geq t$. Let $(I^k)_{k\in\mathbb{N}_0}$ be an increasing sequence in $[0, y/r]$ with $I^0 = 0$ and $I^k \to y/r$ for $k \to \infty$, and let $(\check{p}_0^k)$ be a sequence of beliefs in (p_l^{*k}, p_f^{*k}), where p_l^{*k} and p_f^{*k} are the leader and follower thresholds for investment cost I^k for each k. Further, let $\underline{p}^k$ be given by (10) with $I = I^k$. Since $\underline{p}^k \to 1$ for $k \to \infty$, we have $p_l^{*k} \to 1$ for $k \to \infty$. Thus, the payoff for the leader is at least $v_l^*(\check{p}_0^k) + rI^k \to y$. On the other hand, the best thing that can happen for agent 1 after exiting is that the other agent invests and never exits. Thus, the highest payoff agent 1 can achieve after an exit is $v_f^*(\check{p}_0^k)$ which converges to zero as $I^k \to y/r$. Together it follows that

$$\lim_{k\to\infty} v_l^*(\check{p}_0^k) + rI^k - v_f^*(\check{p}_0^k) = y \tag{11}$$

which implies that there exists a $k^\dagger$, such that for all $k > k^\dagger$, we have

$$v_l^*(\check{p}_0^k) + rI^k > v_f^*(\check{p}_0^k).$$

This inequality implies that a leader cannot gain by exiting at beliefs in the range $[\hat{p}_l^*, p_f^*)$ if $I > I^\dagger$, for some $I^\dagger > 0$. From (11), it follows that $I^\dagger < y/r$. Given that the leader does not exit, there cannot be any $\check{p}_t \in [\hat{p}_l^*, p_f^*)$ at which both agents invest immediately since either agent would prefer delaying the investment and become a follower. Since there is no equilibrium with simultaneous investment, in symmetric equilibrium, both agents must choose the same distribution over initial investment times. Because the continuation strategies are unique, there is a unique investment rate, given by (23), that has the property that each agent is willing to randomize. Thus, there is a unique symmetric equilibrium outcome. By Lemma 2, we have $\hat{p}_l^* < \underline{p}$, for I sufficiently large. In this case, agents' expected payoffs from investing are thus negative for $\check{p}_t \leq \hat{p}_l^*$. Therefore, in every equilibrium, both agents refrain from investing/ rescind their respective investment for good in this range. □

Proof of Theorem 3. *Part* (1.): Suppose $p_0 > 0$ and $\rho > 1/2$ such that $p_0(g, g) \geq p_f^*$. We proceed to verify that the following strategies and beliefs are part of an equilibrium. In the first phase, an agent with signal g invests immediately. An agent with signal b invests immediately with probability $\eta \in [0, 1]$. With probability $1-\eta$, he invests at a random time drawn from an exponential distribution with parameter $\beta^*(p_0(b, b))$. In the second phase of the game, a follower with posterior belief p delays investment by $\tau^*(p)$, and the beliefs at t are updated via Bayes' rule whenever possible. A leader with signal g reverses his investment immediately at $t = 0$ (and stays out) if and only if his posterior belief about θ is lower than $\hat{p}_l^*$. Otherwise he stays invested indefinitely. A leader with signal b reverses his investment immediately at $t = 0$ with some probability $1 - \nu \in [0, 1]$. Either agent who remains invested in the second (or third) phase exits after the occurrence of a failure. Unless otherwise stated, beliefs after off-path histories are specified as follows: In any phase, at time t, the non-deviating agent with signal s assigns probability $p_t(b, s)$ to state $\theta = H$. By the same token, after any off-path exit of agent i, agent $-i$ assigns probability 1 to agent i's signal being b. Any off-equilibrium investment of agent i does not affect agent $-i$ belief about agent i's signal. The deviating agent's beliefs do not change as a result of his deviation.

We show that there exist $\eta, \nu \in (0, 1)$ such that the above strategies and associated beliefs characterize a perfect Bayesian equilibrium. Consider first the second phase, taking as given that each type g invests at $t = 0$ with probability 1, and each type b invests at $t = 0$ with probability η and waits with probability $1 - \eta$. As shown in Lemma 1, the function $\tau^*(p)$ is the optimal delay of the follower with posterior p, and thus given the leader stays in the game, a follower cannot gain from deviating in the second phase of the game. If the leader exits immediately in the second phase, the follower cannot influence that decision.

Suppose agent i with signal s invests at time $t = 0$ in the first phase, and the other agent $-i$ does not, so that at $t = 0$ in the second phase, agent i is the leader and $-i$ the follower. If both expect that the other agent follows the strategy described in the previous paragraph, then the posterior belief of agent i with signal s is $p_0(b, s)$, and the posterior belief of type s of agent $-i$ is, by Bayes' rule,

$$\tilde{p}(\eta,s)=\frac{p_0(s)(\rho+\eta(1-\rho))}{p_0(s)(\rho+\eta(1-\rho))+(1-p_0(s))(1-(1-\eta)\rho)}. \tag{12}$$

If $v_l(p_0(b, b), \tau^*(\tilde{p}(\eta, b))) + rI > v_l^*(p_0(b, b)) + rI > 0$, then the continuation payoffs for either type of agent i is positive (since $v_l(p_0, \tau^*(\tilde{p}(\eta\nu, b))) > v_l(p_0(b, b), \tau^*(\tilde{p}(\eta\nu, b))))$, and both types of agent i remain invested for sure. If, on the other hand, $v_l(p_0(b, b), \tau^*(\tilde{p}(\eta, b))) + rI <$

$0 < v_l(p_0(b,b), \tau^*(p_0)) + rI$, then type b of agent i remains in the game with probability $\nu^* \in (0,1)$ solving $v_l(p_0(b,b), \tau^*(\tilde{p}(\eta\nu^*, b))) + rI = 0$.[10] If $v_l(p_0(b,b), \tau^*(p_0)) + rI < 0$, then type b of agent i exits for sure, and type g remains for sure.

We need to show that there exists a value for η such that neither agent can gain by deviating from the specified strategies in the first phase. Denote by $V_\theta(\eta)$ the value of investing at $t = 0$ for an agent in state θ, and let $W_\theta(\eta, \tau)$ be the value of waiting at $t = 0$ when the agent delays investment by τ as follower. Note that here τ refers to the delay of the investment, given that the other agent invests immediately at $t = 0$. When neither agent invests, then each agent is convinced that the other agent's type is bad, so that there is no longer any uncertainty about the other's private information, and the unique symmetric equilibrium under public information with $\check{p}_0 = p_0(b,b)$ is played after that history. Note here that when the state and strategies are given, the payoff is independent of private signals. Note also that for a follower in the second phase, the optimal delay for type b is $\tau^*(\tilde{p}(\eta, b))$ when type b of the other agent invests with probability η. We write

$$\mathbb{E}[W_\theta^*(\eta)|s] := \max_\tau \mathbb{E}[W_\theta(\eta,\tau)|s] = \mathbb{E}[W_\theta(\eta, \tau^*(\tilde{p}(\eta, s))|s].$$

for the payoff from waiting when using the optimal delay.

There is a pooling equilibrium, i.e. $\eta = 1$, if and only if $p_0(b) \geq p_f^*$, since, in this case, each type of each agent is willing to invest immediately, if the other agent invests for sure, and thus reveals no information. Thus, consider the case $p_0(b) < p_f^*$. Note that in this case, we have $\mathbb{E}[W_\theta^*(1)|b] \geq \mathbb{E}[V_\theta(1)|b]$, since bad types always have incentives to wait when the other agent invests with probability one. There are two cases to consider, $\mathbb{E}[W_\theta^*(0)|b] < \mathbb{E}[V_\theta(0)|b]$ and $\mathbb{E}[W_\theta^*(0)|b] \geq \mathbb{E}[V_\theta(0)|b]$.

(i.) Suppose $\mathbb{E}[W_\theta^*(0)|b] < \mathbb{E}[V_\theta(0)|b]$, i.e., an agent with a bad signal prefers to invest immediately in the first phase at time zero, if the other agent invests with zero probability after a bad signal and invests immediately after a good signal. In this case, there exists a partial (or full) pooling equilibrium in which type g always invests while type b invests with probability $\eta^* \in (0,1]$. Note that the functions $\mathbb{E}[W_\theta^*(\eta)|b]$ and $\mathbb{E}[V_\theta(\eta)|b]$ are convex combinations of continuous functions and hence continuous. Thus, there exists an $\eta^* \in (0,1]$ such that $\mathbb{E}[V_\theta(\eta^*) - W_\theta^*(\eta^*)|b] = 0$, so that an agent with type b is indifferent between investing and not investing, given the other agent invests with

[10] Note that $\tilde{p}(\eta\nu^*, b)$ is the posterior belief of type b of agent $-i$ that the state is H, conditional on the joint event that agent i invested in the first phase and remains in the second phase, where $\eta\nu^*$ is the probability that type b of agent i does this.

probability η^* after observing signal b. We shall now verify an agent of type g's incentives to invest. Note that we have the following inequality:

$$\begin{aligned}
0 &= \mathbb{E}[V_\theta(\eta^*) - W^*_\theta(\eta^*)|b] \leq \mathbb{E}[V_\theta(\eta^*) - W_\theta(\eta^*, \tau^*(\tilde{p}(\eta^*, g)))|b] \\
&= p_0(b)\big(V_H(\eta^*) - W_H(\eta^*, \tau^*(\tilde{p}(\eta^*, g))\big) + (1 - p_0(b))\big(V_L(\eta^*) - W_L(\eta^*, \tau^*(\tilde{p}(\eta^*, g)))\big) \\
&\leq p_0(g)\big(V_H(\eta^*) - W_H(\eta^*, \tau^*(\tilde{p}(\eta^*, g))\big) + (1 - p_0(g))\big(V_L(\eta^*) - W_L(\eta^*, \tau^*(\tilde{p}(\eta^*, g)))\big) \\
&= \mathbb{E}[V_\theta(\eta^*) - W^*_\theta(\eta^*)|g],
\end{aligned}$$

where the first inequality follows from the fact that $\mathbb{E}[W^*_\theta(\eta^*)|b] \geq \mathbb{E}[W_\theta(\eta^*, \tau)|b]$ for all $\tau \geq 0$ by definition, and the second inequality from $p_0(g) > p_0(b)$, and from the fact that investing immediately is strictly better than waiting if and only if the state is H (since there is no gain from delay in state H, and no gain from investing in state L).

(ii.) Now, suppose $\mathbb{E}[W^*_\theta(0)|b] \geq \mathbb{E}[V_\theta(0)|b]$, so that agents with signal b prefer to wait if the other agent invests only if his signal is g. For agents with signal g in this case we have

$$\begin{aligned}
\mathbb{E}[V_\theta(0) - W_\theta(0)|g] = q_0(g)\big(v_l(p_0(g, g), 0) - v^*_f(p_0(g, g))\big) \\
+ (1 - q_0(g))\big(0 \vee v^*_l(p_0) - 0 \vee v_l(p_0, \tau^*(p_0(b, b))\big).
\end{aligned}$$

Since $p_0(g, g) \geq p^*_f$ by assumption, we have $v_l(p_0(g, g), 0) - v^*_f(p_0(g, g)) = 0$, and thus $\mathbb{E}[V_\theta(0) - W_\theta(0)|g] \geq 0$. Thus, in this case, we have a fully separating equilibrium in which g-types invest at $t = 0$, whereas b-types do not.

It remains to be shown that, in both cases (i.) and (ii.), if agent i has incentives to invest at time $t = 0$, then he has no incentive subsequently to exit, provided I is large enough. Similarly to the proof of Theorem 2, let $(I^k)_{k\in\mathbb{N}_0}$ be an increasing sequence in $[0, y/r]$ with $I^0 = 0$ and $I^k \to y/r$ for $k \to \infty$, and let (p^k_0, ρ^k) be a sequence of information structures with $p^{*k}_l < p^k_0 < p^{*k}_f < p^k_0(g, g)$, where p^{*k}_l and p^{*k}_f are, respectively, the leader and follower thresholds for investment costs I^k, such that either $\mathbb{E}[W^{*k}_\theta(0)|b] < \mathbb{E}[V^k_\theta(\eta)|b]$ or $\mathbb{E}[W^{*k}_\theta(0)|b] \geq \mathbb{E}[V^k_\theta(\eta)|b]$ for all $k \geq 0$, where $W^{*k}_\theta(\eta), V^k_\theta(\eta)$ denote the follower and leader value for each k (as above). Moreover, let $\tilde{p}^k(\eta, b)$ be the posterior belief given by (12) at step k, and let $\eta^k \in [0, 1]$ be the critical value with the property that (1) for $p_0(b) < p^{*k}_f$, either type b of each agent is indifferent or else $\eta^k = 0$, and (2) for $p_0(b) \geq p^{*k}_f$, we have $\eta^k = 1$.

Finally, let $\underline{p}^k$ be given by (10) with $I = I^k$. Note that, if $s_i = b$, our assumption that i has incentives to invest at time $t = 0$ implies $\tilde{p}^k(\eta^k, b) \geq \underline{p}^k$ for all k. If $s_i = g$, our assumption in the statement of the theorem implies that i's belief after any history is bounded below by $p_0 \geq p_l^{*k} \geq \underline{p}^k$.

(i.) *Both agents invested.* Note that $\tilde{p}^k(\eta^k, b) \to 1$ as $\underline{p}^k \to 1$, for $k \to \infty$. Thus, the payoff of remaining invested, for either type of agent, is at least $v_l^*(\tilde{p}^k(\eta^k, b), 0) + rI^k \to y$. On the other hand, the best thing that could happen to an agent after exiting would be for the other agent to reveal his type, to invest and never to exit. Thus, the highest payoff either type of agent i could possibly achieve after an exit is $\mathbb{E}[v_f^*(p_0^k(s_{-i}, g))]$ which converges to zero as $I^k \to y/r$. Together it follows that

$$\lim_{k\to\infty} v_l^*(\tilde{p}^k(\eta^k, b), 0) + rI^k - \mathbb{E}[v_f^*(p_0^k(s_{-i}, g)] = y \tag{13}$$

which implies that there exists a $\tilde{k}_1$, such that for all $k > \tilde{k}_1$, we have

$$v_l^*(\tilde{p}^k(\eta^k, b), 0) + rI^k > \mathbb{E}[v_f^*(p_0^k(s_{-i}, g)].$$

This inequality implies that there exists a threshold $\tilde{I}_1$ such that a leader of either type cannot gain by exiting if $I > \tilde{I}_1$. From (13), it follows that $\tilde{I}_1 < y/r$.

(ii.) *Only one agent invested.* First, we argue that type b of each agent i remains invested if he invests himself and his payoff as leader is positive, i.e., if $p_0^k(b, b) > \hat{p}_l^{*k}$. If $p_0^k(b, b) \in [\hat{p}_l^{*k}, p_l^{*k})$, then exit is clearly not optimal, since after i's exit, agent $-i$ (whose type has become known to be b after he did not invest) will never invest going forward. Thus assume $p_0^k(b, b) \geq p_l^{*k}$. Then $v_l^*(p_0^k(b, b)) + rI^k \geq v_l^*(p_l^{*k}) + rI^k$. Since $\underline{p}^k \to 1$ for $k \to \infty$, it follows from $p_0^k(b, b) \geq \underline{p}^k$ that $p_0^k(b, b) \to 1$ for $k \to \infty$, and therefore, $v_l^*(p_0^k(b, b)) + rI^k \to y$. On the other hand, the best thing that could happen for agent i after exiting is that the other agent invests and never exits. Thus, the highest payoff agent i can achieve after an exit is at most $v_f^*(p_t^k(b, b))$ which converges to zero as $I^k \to y/r$. Together it follows that

$$\lim_{k\to\infty} v_l^*(p_t^k(b, b)) + rI^k - v_f^*(\tilde{p}^k(\eta^k, b)) = y \tag{14}$$

which implies that there exists a $\tilde{k}_0$, such that for all $k > \tilde{k}_0$, we have

$$v_l^*(p_0^k(b,b)) + rI^k > v_f^*(\tilde{p}^k(\eta^k, b)).$$

This inequality implies that there exists a threshold $\tilde{I}_0$ such that a leader cannot gain by exiting if $I > \tilde{I}_0$. From (14), it follows that $\tilde{I}_0 < y/r$.

Part (2.): Consider symmetric strategies with the following properties. Each agent with signal g invests with probability ν, and each agent with signal b waits indefinitely in the first phase. If $p_0 \geq \hat{p}_l^*$, an agent who invested at $t = 0$ always remains invested until an accident occurs; if $p_0 < \hat{p}_l^*$, an agent i who invested at $t = 0$ in the first phase rescinds his investment at $t = 0$ in the third phase

if and only if $-i$ did not invest in the second phase at $t = 0$. If agent i invests immediately, then agent $-i$ invests without delay if his signal is g. If agent $-i$'s signal is b, then he delays his investment by $\tau^*(p_0)$. Agents never invests after any other history. Each agent with signal g is indifferent between investing and delaying his investment if

$$q_0(g)v_l(p_0(g,g),0) + (1 - q_0(g))(\max\{v_l^*(p_0) + rI, 0\} - rI) = \nu q_0(g)v_f(p_0(g,g),0).$$

which is equivalent to

$$\nu^* = 1 + \left(\frac{1 - q_0(g)}{q_0(g)}\right)\left(\frac{\max\{v_l^*(p_0) + rI, 0\} - rI}{v_f(p_0(g,g),0)}\right) \tag{15}$$

When agent i with signal g invests, his continuation strategy is optimal by construction. To show that it is optimal for agents with signal b to wait, denote by $V_\theta(\nu)$ the value of investing at $t = 0$ for an agent in state θ, and let $W_\theta(\nu, \tau)$ be the value of waiting at $t = 0$ when the agent delays investment by τ as follower. Let further $W_\theta^*(\nu)$ denote the value of waiting with optimal delay of the follower. By construction of ν^* and τ^*, we have

$$\begin{aligned}
0 &= \mathbb{E}[V_\theta(\nu^*) - W_\theta^*(\nu^*)|g] \\
&= p_0(g)\big(V_H(\nu^*) - W_H(\nu^*, \tau^*(p_0(g,g)))\big) + (1 - p_0(g))\big(V_L(\nu^*) - W_L(\nu^*, \tau^*(p_0(g,g)))\big) \\
&\geq p_0(b)\big(V_H(\nu^*) - W_H(\nu^*, \tau^*(p_0(g,g)))\big) + (1 - p_0(b))\big(V_L(\nu^*) - W_L(\nu^*, \tau^*(p_0(g,g)))\big) \\
&= \mathbb{E}[V_\theta(\nu^*) - W_\theta(\nu^*, \tau^*(p_0(g,g)))|b] \\
&\geq \mathbb{E}[V_\theta(\nu^*) - W_\theta^*(\nu^*)|b].
\end{aligned}$$

Thus, for an agent with signal b, it is a best response to wait. A similar argument to before establishes that it is never optimal for an agent who invested to exit.

Part (3.): Suppose $p_0 > 0$ and $\rho \in (0.5, 1)$ are such that $p_0(g,g) < p_f^*$. The strategies outlined in the theorem imply that an agent who invests in the first phase and becomes the leader reveals himself to be of type g. Suppose that after agent i invests at time t in the first phase, the agents use the following continuation strategy:

- Agent $-i$ with signal g invests at time $t + \Delta$, where $\Delta = \tau^*(p_0(g,g))$.
- If $v_l^*(p_{t+\Delta}) + rI \geq 0$, where

$$p_{t+\Delta} = \frac{p_0}{p_0 + (1-p_0)e^{-\gamma\Delta}},$$

  then type g of agent i remains in the game indefinitely, and each type s of agent $-i$ invests with delay $\tau^*(p_0(s,g))$

- If $v_l^*(p_{t+\Delta}) + rI < 0 < v_l(p_{t+\Delta}, 0)$, then type g of agent i remains for sure until $t + \Delta$ and type g of the follower enters after delay Δ. Beginning at time $t + \Delta$, type b of agent $-i$ invests at rate $\phi_f(s)$ solving

$$0 = y - (1-p_s)\gamma c + \phi_f(s)(v_l(p_s, 0) + rI)$$

  and type g of agent i exits at rate $\phi_l(s)$ solving

$$v_l(p_s, 0) = (1 - rdt - (1-p)\gamma dt - \phi_l(s)dt)v_l(p_{s+dt}, 0).$$

  The exit and investment rates ϕ_l, ϕ_f are defined in a way that the leader and follower are willing to randomize. Note that since delay is profitable for the follower for all $p < p_f^*$, we have

$$v_l(p_s, 0) < (1 - rdt - \gamma dt)v_l(p_{s+dt}, 0).$$

  and thus $\phi_l > 0$.

- If $v_l(p_{t+\Delta}, 0) + rI, v_l(p_{t+\Delta}, 0) < 0$, then type g of agent i remains in until $t + \Delta$ if agent $-i$ invests with delay Δ, and agent i exits otherwise. Type g of agent $-i$ invests with delay Δ, and type b of agent $-i$ never invests.

If agent i with signal g, who invested at some time $t < t^*$, deviates by exiting, we assume that agent $-i$ delays investment indefinitely while both agents are out, yet ignores the deviation completely as soon as agent i re-enters, making it a best response for the deviating agent to re-invest immediately upon exiting (since it was optimal for him to enter in the first place). Given agent i re-invests immediately, it is a best response for agent $-i$ to stay out.[11]

(i) *Derivation of the equilibrium investment rate of agents with signal* g. Henceforth, for each of the three cases above, denote by $V_\theta(s_i, s_{-i})$ the value of agent i conditional on (1) state θ and (2) agent i with signal s_i being the leader, and agent $-i$ with signal s_{-i} using the assigned follower strategy. Similarly, let $W_\theta(s_i, s_{-i})$ be the value of becoming the follower. As we have noted in the main text, these payoffs are independent of time in the first phase, since the signal pair (s_i, s_{-i}) encapsulates all information that is exchanged in the first phase, so that $p_t(s_i, s_{-i}) = p_0(s_i, s_{-i})$. Thus, each agent's expected value of becoming the leader is given by

$$U(q_t(g)) = q_t(g)\mathbb{E}[V_\theta(g, g)|s_i = g, s_{-i} = g] + (1 - q_t(g))\mathbb{E}[V_\theta(g, b))|s_i = g, s_{-i} = b].$$

Type g of each agent is willing to randomize if he is indifferent between investing immediately and waiting for another instant. Hence, the value function for type g of the agent must satisfy the indifference condition

$$U(q_t(g)) = \mu_t q_t(g)\mathbb{E}[W_\theta(g, g)|s_i = g, s_{-i} = g]dt + (1 - rdt - \mu_t q_t(g)dt)U(q_{t+dt}(g)). \quad (16)$$

By Ito's Lemma, the indifference condition (16) can be written as

$$U(q_{t+dt}(g)) = U(q_t(g)) + dU(q_t(g))dq_t(g), \quad (17)$$

where by definition of U, we have $dU(q_t(g))/dq_t(g) = \mathbb{E}[V_\theta(g, g)] - \mathbb{E}[V_\theta(g, b)]$. Bayes' rule implies that the posterior belief at $t + dt$ is

$$q_{t+dt}(g) = \frac{q_t(g)(1 - \mu_t dt)}{1 - q_t(g)\mu_t dt}.$$

The differential change in belief is therefore

$$\frac{dq_t(g)}{dt} \equiv \lim_{dt\to 0} \frac{q_{t+dt}(g) - q_t(g)}{dt} = -\mu_t q_t(g)(1 - q_t(g)). \quad (18)$$

If we now substitute equations (17) and (18) in the indifference condition (16) and ignore

[11] Note that, since the follower and the leader have divergent beliefs at a history with a single investment, our definition of symmetry imposes no restrictions after such histories.

higher order terms, we obtain the expression

$$rU(q_t(g)) = \mu_t q_t(g)\mathbb{E}[W_\theta(g,g) - V_\theta(g,g)|s_i = g, s_{-i} = g]. \tag{19}$$

Since $p_0(g,g) < p_f^*$, the right-hand side of this equation is strictly positive. Simplifying and solving the equation for μ_t yields

$$\mu_t = \frac{rU(q_t(g))}{q_t(g)\mathbb{E}[W_\theta(g,g) - V_\theta(g,g)|s_i = g, s_{-i} = g]}. \tag{20}$$

Here, μ_t is the rate of investment for type g of each agent in the symmetric equilibrium at a given belief $q_t(g)$. Note that since $p_0(g,g) < p_f^*$, we have $W_\theta(g,g) > V_\theta(g,g)$, and thus $\mu_t \in [0,\infty)$. (If $p_0(g,g) \geq p_f^*$, then $W_\theta(g,g) \leq V_\theta(g,g)$, and an equilibrium of the type constructed here does not exist.) Substituting this last expression into Equation (18), we obtain the evolution of the posterior $q_t(g)$ in equilibrium:

$$dq_t(g) = -(1 - q_t(g))\frac{rU(q_t(g))}{\mathbb{E}[W_\theta(g,g) - V_\theta(g,g)|s_i = g, s_{-i} = g]}dt. \tag{21}$$

We obtain the equilibrium belief and equilibrium investment rate at each time t by solving Equation (21) with given initial belief q_0. The initial value problem (21) has the unique solution

$$q_t(g) = \frac{e^{-t\beta^*(p_0(g,g))}U(q_0(g)) + (1 - q_0(g))\mathbb{E}[V_\theta(g,b)]}{e^{-t\beta^*(p_0(g,g))}U(q_0(g)) - (1 - q_0(g))\mathbb{E}\left[V_\theta(g,g) - V_\theta(g,b)|s_i = g\right]}. \tag{22}$$

We now substitute $q_t(g)$ into Equation (20) and simplify to obtain the equilibrium rate of investment

$$\mu_t^* = \frac{e^{-t\beta^*(p_0(g,g))}U(q_0(g))}{e^{-t\beta^*(p_0(g,g))}U(q_0(g)) + (1 - q_0)\mathbb{E}[V_\theta(g,b)|s_i = g]}\beta^*(p_0(g,g)). \tag{23}$$

If $r\mathbb{E}[V_\theta(g,b)] > 0$ then the investment rate μ_t^* diverges to $+\infty$ as $t \to t^*$, where

$$t^* = \log\left(1 + \frac{p_0(g,g)}{p_0}\frac{\mathbb{E}[V_\theta(g,g)]}{\mathbb{E}[V_\theta(g,b)]}\right)^{\beta^*(p_0(g,g))}. \tag{24}$$

If, on the other hand, $\mathbb{E}[V_\theta(g,b)] < 0$, then μ_t^* converges to 0 as $t \to \infty$. Thus, $t^* = \infty$.

(ii) *Agents with signal b prefer to wait until t^*.* For agents with signal b, the incremental opportunity cost from waiting is $r\mathbb{E}[V_\theta(b,s)|s_i = b]dt$. The expected incremental gain from waiting for this type is $\mu_t^* q_t(b)\mathbb{E}[W_\theta(b,g) - V_\theta(b,g)|s_i = b]dt$. We show that when agents with signal g invest at rate μ_t^*, then agents with signal b prefer to wait:

$$r\mathbb{E}[V_\theta(b,s)|s_i = b] \leq \mu_t^* q_t(b)\mathbb{E}[W_\theta(b,g) - V_\theta(b,g)|s_i = b, s_{-i} = g]. \tag{25}$$

Because flow values are positive in state H and negative in state L, i.e., $y \geq 0 \geq y - \gamma c$, we have $V_H(s_i, s_{-i}) \geq 0 \geq V_L(s_i, s_{-i})$. Therefore:

$$\begin{aligned}
\mathbb{E}_t[V_\theta(s_i,s_{-i})|s_i = b] &= p_t(b)\mathbb{E}_t[V_H(s_i,s_{-i})|s_i = b] + (1-p_t(b))\mathbb{E}_t[V_L(s_i,s_{-i})|s_i = b] \\
&\leq p_t(g)\mathbb{E}_t[V_H(s_i,s_{-i})|s_i = b] + (1-p_t(g))\mathbb{E}_t[V_L(s_i,s_{-i})|s_i = b] \\
&\leq p_t(g)\mathbb{E}_t[V_H(b,s_{-i})|s_i = g] + (1-p_t(g))\mathbb{E}_t[V_L(b,s_{-i})|s_i = g] \\
&\leq p_t(g)\mathbb{E}_t[V_H(g,s_{-i})|s_i = g] + (1-p_t(g))\mathbb{E}_t[V_L(g,s_{-i})|s_i = g] \\
&= \mathbb{E}_t[V_\theta(s_i,s_{-i})|s_i = g].
\end{aligned}$$

The first inequality follows because $p_t(g) > p_t(b)$. Note that according to our prescribed strategies, agents with signal g invest earlier than agents with signal b, and exit later. Therefore, we have $V_\theta(s_i, g) \geq V_\theta(s_i, b)$. Since $q_t(g) > q_t(b)$, we thus have $\mathbb{E}_t[V_\theta(b, s_{-i})|s_i = g] \geq \mathbb{E}_t[V_\theta(b, s_{-i})|s_i = b]$ for each θ, which explains the second inequality. The last inequality follows because the strategy of type g is constructed to maximize the continuation payoff after investing. Note that

$$\frac{1}{q_t(s_i)}\mathbb{E}[V_\theta(s_i,s_{-i})|s_i] = \frac{p_t(s_i)}{q_t(s_i)}\mathbb{E}[V_H(s_i,s_{-i})|s_i] + \frac{1-p_t(s_i)}{q_t(s_i)}\mathbb{E}[V_L(s_i,s_{-i})|s_i]$$

Because $\rho > 1/2$ and $p_t(g) > p_t(b)$, it follows that

$$\frac{p_t(b)}{q_t(b)} = \frac{p_t(b)}{\rho p_t(b) + (1-\rho)(1-p_t(b))} < \frac{p_t(g)}{\rho p_t(g) + (1-\rho)(1-p_t(g))} = \frac{p_t(g)}{q_t(g)}$$

and similarly,

$$\frac{1-p_t(b)}{q_t(b)} > \frac{1-p_t(g)}{q_t(g)}$$

Combining the previous results, we find that

$$\begin{aligned}\frac{1}{q_t(b)}\mathbb{E}[V_\theta(s_i,s_{-i})|s_i{=}b]&=\frac{p_t(b)}{q_t(b)}\mathbb{E}[V_H(s_i,s_{-i})|s_i{=}b]+\frac{1-p_t(b)}{q_t(b)}\mathbb{E}[V_L(s_i,s_{-i})|s_i{=}b]\\&\leq\frac{p_t(g)}{q_t(g)}\mathbb{E}[V_H(s_i,s_{-i})|s_i{=}b]+\frac{1-p_t(g)}{q_t(g)}\mathbb{E}[V_L(s_i,s_{-i})|s_i{=}b]\\&\leq\frac{p_t(g)}{q_t(g)}\mathbb{E}[V_H(s_i,s_{-i})|s_i{=}g]+\frac{1-p_t(g)}{q_t(g)}\mathbb{E}[V_L(s_i,s_{-i})|s_i{=}g]\\&=\frac{1}{q_t(g)}\mathbb{E}[V_\theta(s_i,s_{-i})|s_i{=}g]\end{aligned}$$

The previous inequalities, together with (19), imply

$$\frac{1}{q_t(b)}r\mathbb{E}[V_\theta(s_i,s_{-i})|s_i=b]\leq\frac{1}{q_t(g)}r\mathbb{E}[V_\theta(s_i,s_{-i})|s_i=g] \tag{26}$$

$$=\mu_t^*\mathbb{E}[W_\theta(g,g)-V_\theta(g,g)|s_i=g,s_{-i}=g]. \tag{27}$$

(iii) *No exit before failure.* If neither agent has invested at time $t>t^*$, then it is common knowledge that both agents have observed bad signals, and thus there is a unique symmetric continuation equilibrium in this case. Suppose agent i invests at $t<t^*$ and subsequently deviates by exiting. As before, we assume that, after such a history, the non-deviating agent stays out forever while both agents are out, and ignores the deviation as soon as the deviator re-enters, making it a best response for the deviating agent immediately to re-invest. This in turn makes it a best response for the first agent to stay out. □

Proof of Theorem 4. Let P be the distribution over signals for given parameters p_0 and ρ. We write $P(s_i)$ for the probability that a given agent's signal is s_i and $P(s_1,s_2)$ for the probability that the pair of signals is (s_1,s_2).

(1.) Suppose $p_0>p_f^*$. Then, choose $\rho^*>1/2$ such that $p_0(b)>p_f^*$ and $p_0(b,b)>p_2^*$. As we show in the proof of Theorem 3, it follows that for all $\rho<\rho^*$, there exists a pooling equilibrium in which each type of each agent invests immediately. By Theorem 1, this equilibrium is efficient. Hence $\tilde{W}\geq E[W_\theta(s_1,s_2)]$.

(2.) Let $p_f^*>p_0>p_l^*$. Choose $\rho^*>1/2$ such that $p_f^*>p_0(g,g)$ and $p_0(b,b)\geq p_l^*$. By Theorem 3, there exists an equilibrium with delayed entry. It follows from arguments in the proof of Theorem 3 that the expected equilibrium value of the good type of each agent is $\mathbb{E}[V_\theta(g,s_{-i})]\geq\mathbb{E}[v_l^*|g]$. The inequality follows from the fact that leaders have the option to exit. By Equation (25), bad types strictly prefer to delay investment at each $t < t^*$. The expected payoff for an agent of type b who deviates by investing before time t^* is bounded below by

$$q_t(b)v_l(p_0, \tau^*(p_0(g,g))) + (1 - q_t(b))v_l^*(p_0(b,b), \tau^*(p_0)) > \mathbb{E}[v_l^*|b].$$

Therefore, the expected social surplus for each agent is

$$\tilde{W} > P(g)\mathbb{E}[v_l^*|g] + P(b)\mathbb{E}[v_l^*|b] = \mathbb{E}[W_\theta(s^1, s^2)].$$

(3.) Let $p_0 = p_f^*$. Since $p_0(g,g) > p_f^*$, each agent with signal g invests immediately. We show that there exists $\rho^* > 1/2$ such that $\tilde{W} > E[W_\theta(s_1, s_2)]$ for all $\rho \in (1/2, \rho^*)$. The equilibrium is with immediate investment, since $p_0(g,g) > p_f^*$. There cannot be a pooling eqilibrium, since $p_0(b) < p_0 = p_f^*$.

The social welfare per agent therefore satisfies the inequality

$$\tilde{W} \geq P(g)\Big[q_0(g)v_l(p_0(g,g),0) + (1-q_0(g))(\eta v_l(p_0,0) + (1-\eta)v_l(p_0,\tau^*(\tilde{p}_\eta(b))))\Big] \\ + P(b)\Big[q_0(b)v_l(p_0,0) + (1-q_0(b))(\eta v_l(p_0(b,b),0) + (1-\eta)v_l(p_0(b,b),\tau^*(p_\eta(b))))\Big].$$

The right-hand side represents the ex-ante expected payoff for an agent who invests immediately after each signal, which is a lower bound for the equilibrium payoff. Note that $P(g)q_0(g) = P(g,g)$, $P(g)(1 - q_0(g)) = P(b)q_0(b) = P(b,g)$ and $P(b)(1 - q_0(b)) = P(b,b)$. Using $q_0(b)p_0 + (1 - q_0(b))p_0(b,b) = p_0(b)$ together with the linearity of $v_l(p,0)$, we can write

$$\tilde{W} \geq P(g,g)v_l(p_0(g,g),0) + P(b,g)v_l(p_0,0) \\ + (P(g,b) + P(b,b))\Big[\eta v_l(p_0(b),0) + (1-\eta)v_l(p_0(b),\tau^*(\tilde{p}(\eta,b))\Big].$$

When signals are public, then, after each realized pair of signals resulting in the posterior belief $\check{p}_0$, each agent's equilibrium payoff is $v_l^*(\check{p}_0)$ (when ρ^* is chosen so that $p_0(b,b) > p_l^*$ for all $\rho \in (1/2, \rho^*)$. Thus, the expected welfare under public information can be written as

$$E[W_\theta(s_1,s_2)]=P(g,g)v_l^*(p_0(g,g))+P(b,g)v_l^*(p_0)$$
$$+(P(g,b)+P(b,b))\Big[q_0(b)v_l^*(p_0)+(1-q_0(b))v_l^*(p_0(b,b))\Big]. \quad (28)$$

Using the definition of τ^* in Lemma 1, we have

$$v_l^*(p) = v_l(p, \tau^*(p)) = py + (1-p)\lambda_1(y - \gamma c)$$
$$+ (1-p)e^{-(r+\gamma)\tau^*(p)}(\lambda_2 - \lambda_1)(y - \gamma c) - rI.$$

Since $p_0 = p_f^*$, we have $\tau^*(p_0) = \tau^*(p_0(g, g)) = 0$. We have that $\tilde{W} > E[W_\theta(s_1, s_2)]$ if

$$\eta v_l(p_0(b), 0)+(1-\eta)v_l(p_0(b), \tau^*(\tilde{p}(\eta, b))$$
$$> q_0(b)v_l(p_0, 0)+(1-q_0(b))v_l^*(p_0(b, b))). \quad (29)$$

We define $\psi(p) := \frac{p}{1-p} = e^{\phi(p)}$ and $\alpha := \frac{r+\gamma}{\gamma} > 1$. Then, the left-hand side of Inequality (29) can be written as

$$\eta v_l(p_0(b), 0) + (1-\eta)v_l(p_0(b), \tau^*(\tilde{p}(\eta, b)) = p_0(b)y + (1 - p_0(b))\lambda_1(y - \gamma c)$$
$$+ (1 - p_0(b))\left[\eta + (1-\eta)\psi(\tilde{p}(\eta, b))^\alpha \psi(p_f^*)^{-\alpha}\right](\lambda_2 - \lambda_1)(y - \gamma c) - rI \quad (30)$$

From Bayes' rule and the definition of ψ, we have

$$\psi(p_0(b))=\frac{p_0}{1-p_0}\frac{1-\rho}{\rho}=\psi(p_0)/\psi(\rho), \quad \psi(\tilde{p}(\eta,b))=\psi(p_0(b))\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right).$$

If we now use the previous equalities to factor out $\psi(p_0(b))^\alpha \psi(p_f^*)^{-\alpha}$ from the square brackets in (30), we obtain

$$\eta v_l(p_0(b),0)+(1-\eta)v_l(p_0(b),\tau^*(\tilde{p}(\eta,b))=p_0(b)y+(1-p_0(b))\lambda_1(y-\gamma c)$$
$$+(1-p_0(b))\psi(p_0(b))^\alpha\psi(p_f^*)^{-\alpha}\left[\eta\psi(\rho)^\alpha+(1-\eta)\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right)^\alpha\right](\lambda_2-\lambda_1)(y-\gamma c)-rI. \quad (31)$$

The right-hand side of Inequality (29) is given by

$$q_0(b)v_l(p_0,0)+(1-q_0(b))v_l^*(p_0(b,b)) = p_0(b)y + (1-p_0(b))\lambda_1(y-\gamma c)$$
$$+\left[q_0(b)(1-p_0)+(1-q_0(b))(1-p_0(b,b))\psi(p_0(b,b))^{\alpha}\psi(p_f^*)^{-\alpha}\right](\lambda_2-\lambda_1)(y-\gamma c)-rI. \tag{32}$$

From Bayes' rule it follows that

$$\frac{q_0(b)}{1-p_0(b)}=\frac{p_0(b)\rho+(1-p_0(b))(1-\rho)}{1-p_0(b)}=\left(\frac{p_0}{1-p_0}\frac{1-\rho}{\rho}\right)\rho+(1-\rho)=\frac{1-\rho}{1-p_0},$$
$$\frac{1-q_0(b)}{1-p_0(b)}=\frac{p_0(b)(1-\rho)+(1-p_0(b))\rho}{1-p_0(b)}=\left(\frac{p_0}{1-p_0}\frac{1-\rho}{\rho}\right)(1-\rho)+\rho=\frac{\rho}{1-p_0(b,b)}.$$

Using these equalities together with the identity

$$\psi(p_0(b,b)) = \frac{p_0(b)}{1-p_0(b)}\frac{1-\rho}{\rho} = \psi(p_0(b))\psi(1-\rho)$$

to factor out $(1-p_0(b))\psi(p_0(b))^{\alpha}\psi(p_f^*)^{-\alpha}$ from the square brackets in (32), we obtain

$$q_0(b)v_l(p_0,0)+(1-q_0(b))v_l^*(p_0(b,b))=p_0(b)y+(1-p_0(b))\lambda_1(y-\gamma c)$$
$$+(1-p_0(b))\psi(p_0(b))^{\alpha}\psi(p_f^*)^{-\alpha}\left[(1-\rho)\psi(\rho)^{\alpha}+\rho\psi(\rho)^{-\alpha}\right](\lambda_2-\lambda_1)(y-\gamma c)-rI.$$

Define the functions,

$$h(\eta,\rho) := \eta\psi(\rho)^{\alpha} + (1-\eta)\left(\frac{\rho+(1-\rho)\eta}{1-\rho+\rho\eta}\right)^{\alpha}, \quad g(\rho) := (1-\rho)\psi(\rho)^{\alpha} + \rho\psi(\rho)^{-\alpha}.$$

Condition (29) is thus equivalent to $\inf_\eta h(\eta,\rho) > g(\rho)$. One calculates that the partial derivative of h at $\rho = 1/2$ is $\lim_{\rho\to 1/2}\partial_\rho h(\eta,\rho) = 4\alpha\left(2\eta^2-\eta+1\right)/(\eta+1)$. The function $\lim_{\rho\to 1/2}\partial_\rho h(\eta,\rho)$ has its minimum in η at $\sqrt{2}-1$ and is thus larger than $4\left(4\sqrt{2}-5\right) > 0$. On the other hand, $g'(1/2) = 0$. Thus, there exists a $\rho^* > 1/2$ such that for all $\rho \in (1/2,\rho^*)$, we have $\tilde{W} > E[W_\theta(s_1,s_2)]$.

□

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