Do Stronger Patents Lead To Faster Innovation? The Effect Of Clustered Search*

Kaustav Das[†] Nicolas Klein[‡]

December 28, 2022

Abstract

We analyse a model of two firms that are engaged in a patent race. Firms have to choose in continuous time between an *established* and an *innovative* method of pursuing a decisive breakthrough. They share a common belief about the likelihood of the innovative method being good. The unique Markov perfect equilibrium coincides with the cartel solution if and only if firms are symmetric in their abilities of leveraging a good innovative method or there is no patent protection. Otherwise, equilibrium will entail excessive clustering of efforts in the innovative method, as compared to the cartel benchmark, for any level of patent protection. We show that the expected time to a breakthrough is minimised at an interior level of patent protection, providing a novel possible explanation for the decrease in R&D productivity sometimes associated with a greater concentration of research efforts in riskier areas and stronger patent protections.

JEL Classification Numbers: C73, D83, O31.

Keywords: R&D competition, Clustering, Two-armed Bandit, Learning

1 Introduction

Innovative methods are an important driver of success in many industries. Consider the pharmaceutical industry and its quest for a better way of treating Alzheimer's disease, for instance. Alzheimer's

*This paper supersedes three earlier versions entitled respectively "Do Stronger Patents Lead To Faster Innovation? The Effect Of Duplicative Search," "Inefficient Duplicative Search in a Patent Race" and "Excessive Duplicative Search in a Patent Race." We thank Siddhartha Bandyopadhyay, Subir Bose, Dirk Bergemann, Kalyan Chatterjee, Martin Cripps, Sanjeev Goyal, Johannes Hörner, Vijay Krishna, Igor Letina, Arina Nikandrova, Sven Rady, Larry Samuleson and Caroline Thomas for their helpful comments. Thanks are also due to the participants of the Warwick Lunchtime Theory Workshop 2019, the Game Theory Conference at Edinburgh 2019, the Game Theory Symposium at the University of York 2018, the Annual Conference at the Indian Statistical Institute New Delhi 2018.

[†]University of Leicester, School of Business, Brookfield, Leicester LE2 1RQ; Email: daskaustav84@gmail.com

[‡]Corresponding Author: Université de Montréal and CIREQ, Département de sciences économiques, C.P. 6128 succursale Centre-ville Montral, QC H3C 3J7, Canada, Email:kleinnic@yahoo.com, Tel: (+1)514-343-7908

is characterised by both a decrease in acetylcholine (neurotransmitter) levels in the brain and the accumulation of β-amyloid plaques. The current method of treatment is based on the widely marketed drug Donepezil, which increases acetylcholine levels but which can only slow down the progression of the disease without curing it. Research efforts over the past decade, by contrast, have been focussed on finding a drug counter-acting the accumulation of β-amyloid plaques. However, as innovative approaches toward this goal have failed to lead to success, researchers are currently exploring the possibility of designing a drug that would combat the accumulation of β-amyloid plaques via an increase in neurotransmitter levels.[1] Indeed, there is some evidence that Donepezil has a beneficial effect on the level of β-amyloid plaques.[2]

When firms search for success using an innovative method, their competition entails a positive informational externality, besides the payoff externality that is typical for patent races. The importance of the latter depends on the level of patent protection afforded by the legal system. Concurrently, the fact that a competitor has been unsuccessfully looking for a breakthrough using a particular method is useful information to the firm, as it will inform its optimal future R&D choices. In our Alzheimer's example, failed clinical trials by a pharmaceutical company indeed provide crucial insights that also help shape competitors' future research efforts.

In this paper, we study process innovation[3] in a setting in which two firms are engaged in a patent race and their research choices are observable, using a variant of the two-armed *exponential-bandits* framework of Keller, Rady and Cripps (2005). In our baseline model, we take the *scale* of R&D as given, analysing the problem of allocating a *given* resource flow among two competing methods methods of R&D. More specifically, there is an established work method either firm can use, which leads to a success at the first jumping time of a Poisson process with a known rate. As Donepezil is already known to have an effect on β-amyloid plaques, this would correspond to the search for a drug that seeks to fight the concentration of β-amyloid plaques by increasing neurotransmitter levels. Both firms also have access to an innovative work method that is either good or bad. Whether it is good or bad is initially unknown to the firms, who share a common initial belief about it. If the innovative method is good, it leads to a success at a faster rate than the established method. We allow for one firm to be more efficient than the other in its exploration of the innovative method, achieving a success more quickly conditionally on the method being good. If the innovative method is bad, it never yields a success. The first success ends the game, yielding a payoff that is shared between the firms as a function of the level of patent protection that prevails. Both firms discount future payoffs at a common rate.

The innovative method is good for one firm if and only if it is good for the other firm as well. As

[1]See Moss (2018).

[2]See Dong et al. (2009).

[3]While the phrase *process innovation* is often used to refer to innovations that decrease the cost of production of a certain good, we here use the term to signify the use of a new method to achieve a certain goal.

either firm's actions are perfectly publicly observable, the R&D race between the two firms involves a positive informational externality. Indeed, the longer the innovative method is unsuccessfully tried by *either* firm, the more pessimistic *both* firms become about its quality. There is, however, also a (negative) payoff externality between the firms, the strength of which depends on the level of patent protection afforded to the winner of the R&D race.

We first analyse the problem of a cartel that endeavours to maximise the aggregate expected discounted payoff. In the optimal cartel solution, the stronger firm experiments with the innovative method if and only if the firms' belief that the innovative method is good exceeds its myopic cutoff; i.e., if and only if the *instantaneous* expected arrival rate of a success is higher with the innovative method than the established one for the stronger firm. If the other firm is equally productive with the innovative method, it behaves exactly as the first firm in the cartel solution. Otherwise, the less productive firm anticipates that the more productive firm will continue exploring the innovative method until its myopic cutoff is reached. At its own myopic cutoff, the less productive firm thus reasons that, if it goes on experimenting a bit longer, the more productive firm's myopic cutoff is reached sooner (conditionally on no success); put differently, the amount of time the productive firm will henceforth spend on exploring the innovative method is reduced. Based on the current belief, this means that the overall likelihood of a success by the more productive firm decreases. Since, at the less productive firm's myopic cutoff, its own expected breakthrough rate is exactly equalized between the two methods, this explains why the cartel will apply a cutoff more optimistic than its myopic cutoff to the less productive firm. As the expected time to a breakthrough is minimised when both firms switch methods at their respective myopic cutoffs, this implies that a profit-maximising cartel will delay the expected breakthrough time by making the weaker firm give up on the innovative method prematurely.

We go on to show that, for any level of patent protection, our game admits a *unique Markov perfect equilibrium*, with the firms' common belief that the innovative method is good as the state variable. In contrast to the case of pure informational externalities (see Keller, Rady and Cripps (2005)), our unique equilibrium is *always in cutoff strategies*. If and only if both firms are equally productive with a good innovative work method or there is no patent protection whatsoever, the unique Markov perfect equilibrium coincides with the cartel solution. In the case of symmetric firms, it also minimises the expected time to a breakthrough.

By contrast, if one of the firms happens to be more productive with a good innovative method, e.g., because it has a bigger or better research or production department,[4] the unique Markov perfect equilibrium leads to excessive clustering of innovative efforts, as compared to the cartel solution. The stronger firm always acts as in the cartel benchmark, being the last to give up experimenting with the innovative method in equilibrium. The less productive firm, by contrast, trades off two effects: on the one hand, because of discounting, it wishes for a breakthrough to occur as quickly as possible,

[4]We assume that the established method has been around long enough that both firms are equally productive with it.

while, on the other hand, it would prefer that it was the one achieving the breakthrough, rather than its competitor. Which of these effects prevails at its myopic belief threshold depends on the strength of the patent protection afforded to the first innovator. If patent protection, and thus the firms' payoff rivalry, is strong, the above reasoning explains why the less productive firm will endeavour to "eat up" some of the stronger firm's comparative advantage, by extending experimentation below its myopic cutoff, thereby reducing the expected time the stronger firm will spend on the innovative method. This implies that, for strong patent protections, research productivity, as measured by the expected time to a breakthrough, is *lowered* because the weaker firm extends research on the innovative method beyond its myopic threshold.

For weaker levels of patent protection, by contrast, the weaker firm will stop experimentation at a threshold exceeding its myopic cutoff, while still extending it beyond the cartel threshold. Thus, while the effect is mitigated as compared to the cartel, research productivity is still suboptimal because the weaker firm gives up on the innovative method too quickly. There is a unique, interior, level of patent protection that makes the weaker firm behave in equilibrium exactly as though it were myopic; this level of patent protection minimises the expected time to a breakthrough.

In summary, the cartel asks the weaker firm to step aside somewhat for the stronger firm in the cooperative solution. In equilibrium, by contrast, the weaker firm always stands aside less. If patent protection is strong, it is willing to incur a reduction in its own breakthrough rate by extending experimentation below its myopic threshold, in order to discourage the stronger firm by its hapless experimentation. If patent protection is weak, the weaker firm is incurring a reduction in its own breakthrough rate by switching to the established method at a threshold above its myopic cutoff, in order to encourage the stronger firm to spend more time exploring the innovative method.

In Section 6, we show that the *intertemporal-substitution effect* we identify in our baseline model is robust to various extensions, which make our setup more realistic. Thus, we explicitly introduce a post-innovation product market (Subsection 6.1), R& D costs (Subsections 6.2 and 6.3), and allow for a breakthrough via the innovative method to be more valuable than a breakthrough that is achieved via the established method (Subsection 6.4). While our qualitative results are unchanged by the explicit introduction of the product market and small research cost, larger research costs give rise to familiar trade-offs: If R& D costs are in the form of flow costs (Subsection 6.2), our equilibrium requires patent protection to be strong enough for the firm to be willing to incur the expenditure. If, however, research costs are in the form of an upfront fixed cost (Subsection 6.3), patent protection has to be *weak* enough, for the spoils from a success by the stronger firm to be important enough, to entice the weaker firm to enter the fray. If a success by the innovative method is more valuable (Subsection 6.4), the existence of our equilibrium requires firms to be very asymmetric.

Pfizer has pulled out from Alzheimer's drug research in January 2018 while its competitors keep pursuing it, which suggests that heterogeneity among firms is indeed a feature of real-world R&D

races. Analysing a large database that contains information on R&D projects for more than $28,000$ cases, Pammoli, Magazzini and Riccaboni (2011) conclude that, in the period 1990-2010, there has been a decline in R&D productivity in pharmaceuticals, which cannot be fully explained by the market forces of demand and competition. Simultaneously, they observe an increasing concentration of R&D investments in relatively more risky areas. Incidentally, this time period coincides with the implementation of the Agreement on Trade Related Intellectual Property Rights (TRIPS), which covers pharmaceutical products or processes invented since January 1, 1995. This agreement obliges all WTO members to afford patent protections for pharmaceutical inventions. Previously to the TRIPS agreement, copies of medicines that were patent-protected elsewhere were often widely available in many developing countries.[5] TRIPS has thus significantly strengthened patent protections in the pharmaceutical sector. While Aghion et al. (2005) show an inverted U-relationship between competition and innovation, their mechanism, which relies on the differential effect of competition on pre-innovation and post-innovation rents, does not account for different levels of project riskiness. To the best of our knowledge, we are the first to propose an explanation for the combination of a decline in R&D productivity, a strengthening of the patent regime and an increasing concentration of research efforts in riskier areas.

A decrease in pharmaceutical R&D productivity connected with stronger patent protections has been noted elsewhere. Indeed, studying pharmaceutical patent protection for the time period 1978–2002, Qian (2007) writes that “there appears to be an optimal level of intellectual property rights regulation above which further enhancement reduces innovative activities” (see his Abstract). He goes on to note: “National patent laws would also induce domestic investors to switch from imitative activities to innovative ones” (see Qian (2007), p. 437). Sakakibara and Branstetter (2001) likewise find no evidence of an increase in R&D output subsequent to a strengthening of patent projections in Japan in 1988.[6] Galasso and Schankerman (2015) and Sampat and Williams (2019) analyze the impact of patent protections on follow-on innovations. While the former show that the impact depends on the field of activity, the latter conclude that, on average, patents on genes have had no important effects on follow-on innovations. Our model formalises the idea that, in an R&D race with observable actions, excessive clustering in riskier innovative efforts may be caused by strong patent protections.

The rest of the paper is organised as follows. Section 2 reviews the relevant literature, Section 3 describes the environment, while Sections 4 and 5 describe the analysis with symmetric and asymmetric players respectively. Section 6 explores extensions to our baseline model, and Section 7 concludes. The Appendix contains the derivation of the payoff functions and formal proofs of our results.

[5] See e.g.Boulet et al. (2000).

[6] They find no evidence of an increase in R&D expenditure either. This is consistent with our model if we interpret investment in the established method as R&D expenditure as well. Bessen and Hunt (2004) also find that R&D intensity in the US software industry decreased subsequently to an enhancement of patent protections for computer programs that occurred in the US during the 1980s and 1990s by virtue of a gradual evolution of the appertaining jurisprudence. Our baseline model only pertains to the choice of methodology in R&D races, rather than the level of R&D investments.

2 Related Literature

Our paper contributes to the literature exploring the link between patent strength and R&D investment. Bessen and Maskin (2009) assumes sequential and complementary innovations, and provide a theoretical explanation for a negative link between patent strength and R&D investment. Chen et al. (2014) considers a discrete-time model with an infinite horizon. There are two potentially innovating firms, an incumbent who made an innovation earlier, and an entrant who is conducting R&D. If the entrant makes a discovery, then profits in the current period are shared according to the patent protection level, and, next period, the entrant and the incumbent swap their roles. Stronger patent protection favours the incumbent. Thus, patent protection has two countervailing effects on investments in R&D: It encourages (discourages) R&D by increasing future profits (decreasing profits at the period of one's own breakthrough) from a discovery, the relative importance of these effects depending on the discount factor. O'Donoghue et al. (1998) considers a setting where there is a continuum of firms to which innovations arrive at the jumping times of a known Poisson process. A patent is defined by its expiration date as well as by its *lagging* and *leading* breadth, with the former providing protection from imitation, and the latter from improved innovations. They show that if there is no leading breadth, the equilibrium rate of innovation is too low. For patents with infinite leading breadth, the optimal investment rate is achieved, or even exceeded, as patent duration goes to infinity. Gangopadhyay and Mondal (2012) point out that strong patents can harm subsequent innovation, which builds on previous knowledge, by making this previous knowledge less accessible. None of these papers focuses on the effect of patent strength on innovation when there are different research avenues, which is a central feature of our model. The closest paper with this feature is Chen et al. (2018). In their framework, innovation can be achieved either through a safe or a risky method. Innovation by the safe method always leads to an improvement that is higher than the patentability threshold; innovation by the risky method leads to a stochastic improvement, which may or may not exceed the patentability standard, which is chosen by the government. In their model, there is no learning over time about the stochastic process governing the risky research avenue. They show that *weak* patents, i.e., a low patentability standard, distort research towards the risky option. In our paper, by contrast, we incorporate learning over time about the stochastic process governing the risky, innovative, avenue, and show that *strong* patents distort research in the direction of the innovative method.

The problem of incentivizing a single agent to engage in innovation has been analysed by Chen and Ishida (2018) and Klein (2016) in continuous time and by Manso (2011) in a two-period model. Of this latter setting, Ederer (2013) studies an extension to two agents. Schneider and Wolf (2019) analyses the problem of a single agent who faces a deadline to solve a given problem and dynamically chooses between an innovative, and potentially quick, method and an established, slower, method. Chen et al. (2020) analyses the signalling problem of an agent whose payoff depends on the market's belief about his type and who needs to decide when irreversibly to stop pursuing a risky project, which can be either good or bad. In their model, agents differ in both their ability to identify, and their ability to implement, a good project.

Our model builds on the literature on strategic experimentation with bandits, started by Bolton and Harris (1999). In particular, we use a variant of the exponential model of Keller, Rady and Cripps (2005). We deviate from Keller, Rady and Cripps (2005)'s classical model in three respects: (i) we introduce payoff externalities and (ii) asymmetric arrival rates, while, in our baseline model, (iii) players do not have access to a safe arm, whose payoff is deterministic and independent of the other player's behavior. These changes lead to a dramatic change in predictions: (1.) While in Keller, Rady and Cripps (2005), there is a continuum of MPEs, our model features a unique MPE; (2.) while there does not exist an equilibrium in cutoff strategies in Keller, Rady and Cripps (2005), in the baseline model, our unique equilibrium is in cutoff strategies. These differences obtain even if arrival rates are the same, i.e., if we make only changes (i) and (iii). Finally, while all equilibria in Keller, Rady and Cripps (2005) have the property that players experiment too little as compared to the cartel solution, our equilibrium coincides with the cartel solution for symmetric arrival rates, and, with the incorporation of change (ii), our equilibrium features strictly too much experimentation if there is some positive, though arbitrarily weak, patent protection.

Das, Klein and Schmid (2019) has made change (ii) only to Keller, Rady and Cripps (2005)'s model, introducing asymmetric Poisson arrival rates. There, we show that there exists an MPE in cutoff strategies if and only if the asymmetry is stark enough. There always exists a continuum of other MPE, which are, however, welfare-dominated by the cutoff equilibrium if it exists.

Besanko and Wu (2013) makes change (i) only to Keller, Rady and Cripps (2005)'s model, introducing payoff externalities. They focus on symmetric MPE in their analysis, and characterise the unique equilibrium in that space. While our unique Markov perfect equilibrium, which is symmetric, is efficient for symmetric players, Besanko and Wu (2013)'s unique symmetric Markov perfect equilibrium features over-experimentation in the case of a negative payoff externality. In our model, by contrast, in which the players' "safe" option consists of a *process* for a given project rather than an alternative, safe, *project* (change (iii)), the negative payoff externality leads to equilibrium over-experimentation with respect to the cartel solution if and only if we make change (ii) as well.

Akcigit and Liu (2016) also analyses a variant of Keller, Rady and Cripps (2005)'s two-armed bandit model with one risky and one safe method of investigation, while assuming players cannot return to an arm they have previously discarded. The negative payoff externality here is akin to that in the treasure-hunt game of Chatterjee and Evans (2004), which is the first paper to analyse project choice in a dynamic winner-takes-all competition. For the case of costs that are asymmetric across research avenues, they show that there is either *too much* or *too little* exploration of a research avenue, depending on the players' prior belief. In a model without informational externalities, Wong (2018) analyses an R&D race where firms choose when irreversibly to exit and innovation can occur through a technology of initially unknown quality only; this quality is independent across firms.

Halac, Kartik and Liu (2016) characterises sharing and disclosure rules in contests that maximise the probability of an innovation, the feasibility of which is ex-ante uncertain. Bimpikis, Ehsani and Mostagir (2019) considers two-stage contests and analyses the mechanisms for disclosure of a first-stage breakthrough a designer will want to commit to. In our baseline model, we, by contrast, abstract from both private information and the question of effort provision to focus on firms' choice of method; moreover, a breakthrough consists of a single successful stage.

Our paper also contributes to the relatively less explored area of choice of methodological approach in R&D races.[7] Indeed, we show that, when firms are asymmetric and there is *some*, however modest, level of patent protection, there is always some excessive clustering of innovative research effort compared to the cartel optimum. Whether there is excessive or insufficient clustering compared to the optimum of a social planner who wants to speed up as much as possible the arrival of a breakthrough depends on the strength of the patent regime. In a static model with winner-takes-all competition, Klette and de Meza (1986) allows firms to choose the riskiness of R&D strategies. They find that, if invention times are symmetrically distributed, the market equilibrium entails riskier R&D strategies than the social optimum. Also analysing a static model with winner-takes-all competition, Bhattacharya and Mookerjee (1986) shows that, when firms are symmetric and not excessively risk-averse, market allocations and socially optimal allocations coincide, both requiring extreme specialisation. However, with sufficient risk aversion, there is a tendency towards insufficient clustering. Dasgupta and Maskin (1987), by contrast, assume that a project is the costlier the more unusual it is, and find that market research portfolios consist of projects that are too highly correlated. Choi and Gerlach (2014) considers the choice between an easier and a harder project, which are complementary. In contrast to our setting, the success probabilities for each project are common knowledge. They show that, in equilibrium, there tends to be excessive clustering of efforts in the easier project. Brian and Lemus (2017) also analyses the choice of research project, when the success probabilities for each project are common knowledge. In order to guarantee that firms behave in a socially optimal way in Markov perfect equilibrium in their model, Brian and Lemus (2017) shows that it is necessary that the transfer policy condition on the properties of projects that are not discovered along the path of play. Letina (2016) analyses a static model where N symmetric firms compete in the pre-innovation market by choosing a subset from a continuum of different research projects. All approaches are initially equally likely to succeed and it is known that exactly one of them will. There is clustered equilibrium effort in projects with lower costs, with fewer firms developing the more expensive approaches.

[7] As mentioned above, our baseline model abstracts from the choice of the *scale* of R&D, to focus our analysis on the allocation of a *given* resource among the various methods of R&D. The issue of choosing the scale of R&D is well documented in the literature (see Lee and Wilde (1980); Reinganum (1982)).

3 The Environment

Two firms are simultaneously trying to achieve a breakthrough in continuous time. The first breakthrough yields a payoff of α to the firm accomplishing it, and $(1-\alpha)$ to the competing firm, where $\alpha \in [\frac{1}{2}, 1]$. We interpret the parameter α as measuring the strength of patent protection afforded to the firm achieving the first breakthrough. Indeed, $\alpha > \frac{1}{2}$ implies that the firm accomplishing the first breakthrough gets a premium, with $\alpha = 1$ corresponding to the *winner-takes-all* case; i.e., the first firm to innovate appropriates all the rent. There are two work methods the firms can adopt to achieve the breakthrough. One method, method S, is *established* (*safe*) in that it yields a breakthrough at the first jumping time of a Poisson process with known intensity $\lambda_0 > 0$. The other method, method R, is *innovative* (*risky*), in that it is not initially known if it is good or bad, its quality being the same for both firms. If it is good, it produces a breakthrough for firm $i \in \{1, 2\}$ at the first jumping time of a Poisson process with intensity $\lambda_i > \lambda_0$. If it is bad, it never yields a breakthrough for either firm. We assume $\lambda_1 \geq \lambda_2$; i.e., conditionally on the innovative method being good, firm 1 will achieve the breakthrough weakly faster in expectation. For the rest of the paper, the *established* method will be denoted S and the *innovative* method will be denoted R. Both firms discount the future using the common discount rate $r > 0$. Firms do not incur any direct costs for adopting either method. They share a common prior belief $p \in (0, 1)$ that method R is good. Firms' choices of methods are perfectly publicly observable. This implies that, at any time point, firms will also share a common posterior belief.

Evolution of beliefs: If $k_{i,t}$ is an indicator variable for firm i adopting the innovative method, then conditionally on no success arriving via the innovative method, the common posterior p_t evolves a.s. according to

$$dp_t = -(k_{1,t}\lambda_1 + k_{2,t}\lambda_2)p_t(1-p_t)\,dt.$$

Welfare

We have motivated our paper by R& D in the pharmaceutical sector. It thus seems reasonable to assume that a social planner may want to speed up as much as possible the arrival time of a breakthrough, such as a cure for Alzheimer's disease. We therefore adopt the social welfare function

$$W = \frac{TS}{\tau} \tag{1}$$

where TS is the post-innovation total surplus (i.e., the sum of consumer and producer surplus), and τ denotes the expected time to first breakthrough. In our baseline specification, TS is independent of the level of patent protection α and hence to maximise W, the social planner should minimise τ.

In contrast, we define the cartel's problem as that of seeking to maximise the sum of the firms' discounted payoffs. As we will see below, unless firms are symmetric, the solution to the cartel's problem does not coincide with the social-welfare optimum.

4 Symmetric Firms

In this section, we analyse the case of firms that are symmetric in their ability to achieve a success by a good innovative method. This means that we have $\lambda_1 = \lambda_2 > \lambda_0$. We first analyse the cartel's problem.

4.1 The cartel's problem

Without loss of generality, we can restrict the cartel to Markov strategies $k(p_t)$ with the posterior belief p_t as the state variable, where k denotes the number of firms the cartel assigns to method R.[8] This implies $k(p_t) \in \{0,1,2\}$. Let $v(p)$ be the value function of the cartel. Then we have

$$rv = \max_{k \in \{0,1,2\}} \{2\lambda_0(1-v) + k[\lambda_1 p \left(1 - v - (1-p)v^{'}\right) - \lambda_0 (1-v)]\}$$

The expression $2\lambda_0(1-v)$ denotes the expected flow payoff the cartel can guarantee itself by assigning both firms to method S. The expression $\lambda_1 p \left(1 - v - (1-p)v^{'}\right) - \lambda_0 (1-v)$ reflects the premium the cartel gets by assigning an additional firm to method R. Note that, by linearity, even if firms' efforts were divisible, it would be without loss for the cartel to choose $\{k(p_t)\}_{t \geq 0}$ with $k(p_t) \in \{0,2\}$ for all $t \geq 0$. The cartel's solution is described in the following proposition. It shows that maximisation of joint profits requires players to choose the myopically optimal method. To state the proposition, we use the function $\mu(p)$ defined in Appendix A. In the symmetric case, $\mu(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{2\lambda_1}}$.

Proposition 1 *The cartel's optimal policy $k^*(p)$ is given by*

$$k^*(p) = \begin{cases} 2 & \text{if } p \in (p_1^*, 1] \\ 0 & \text{if } p \in [0, p_1^*] \end{cases},$$

where $p_1^* = \frac{\lambda_0}{\lambda_1}$. The cartel's value function is given by

$$v(p) = \begin{cases} \frac{2\lambda_1 p}{r+2\lambda_1} + \frac{4\lambda_0(\lambda_1-\lambda_0)\mu(p)}{(r+2\lambda_0)(r+2\lambda_1)\mu(p_1^*)} & \text{if } p \in (p_1^*, 1] \\ \frac{2\lambda_0}{r+2\lambda_0} & \text{if } p \in [0, p_1^*] \end{cases}$$

[8] We suppress the arguments whenever this is convenient.

Proof. Proof is by a standard verification argument. Please refer to Appendix B for details, and Appendix A.1 for the ODE satisfied by the cartel's value. ■

Note that, in the symmetric case, the cartel's policy minimises the expected time to a breakthrough. In the next subsection, we analyse the non-cooperative game between the firms.

4.2 Non-cooperative game

We restrict ourselves to Markov perfect equilibria, with the firms' common belief as the state variable. A Markov strategy for player i ($i = 1, 2$) is defined as a left-continuous function $k_i : [0,1] \to \{0,1\}$, $p \mapsto k_i(p)$. Let v_i be the value function of player i. Given k_j ($j \neq i$), player i's Bellman equation is

$$\begin{aligned} rv_i = \max_{k_i \in \{0,1\}} & \lambda_0[1-2v_i] + k_i\{\lambda_1 p[\alpha - v_i - (1-p)v_i^{'}] - \lambda_0[\alpha - v_i]\} \\ & + k_j\{\lambda_1 p[(1-\alpha) - v_i - (1-p)v_i^{'}] - \lambda_0[(1-\alpha) - v_i]\}. \end{aligned} \quad (2)$$

From this Bellman equation, we can derive the best responses of the firms, using the ODEs exhibited in Appendix A.1.

Suppose firm $j \neq i$ is adopting method R at the belief $p \in (0,1)$. By left-continuity, there is a left-neighbourhood of p in which j is adopting R. If i best-responds to j by adopting R in some subset of this left-neighbourhood, its value function satisfies

$$v_i \geq \frac{2\lambda_0\alpha + \lambda_1 p(1-2\alpha)}{r + 2\lambda_0}$$

on this subset. If the inequality is strict, adopting R is i's unique best response. By the same token, if the other firm is adopting the method S in some left-neighbourhood of p, then, if firm i best-responds by adopting the method R, its value function satisfies

$$v_i \geq \frac{\lambda_0}{r + 2\lambda_0};$$

if the inequality is strict, adopting R is i's unique best response.

These simple observations allow us to prove the following result, which shows that the unique MPE in this setting coincides with the cartel's solution. This situation is depicted in Figure 1.

Proposition 2 *If firms are symmetric, the unique MPE coincides with the cartel's solution (and thus minimises the expected time to a breakthrough), for any level of patent protection* $\alpha \in [\frac{1}{2}, 1]$.

Proof. See Appendix C. ■

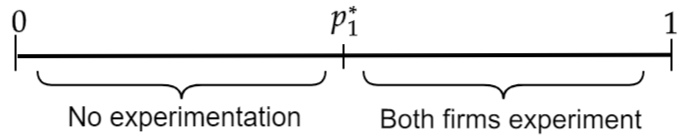

Figure 1: Symmetric Firms

5 Asymmetric Firms

In this section, we analyse the situation of firms that differ in their abilities to achieve a success by a good innovative method, i.e., $\lambda_1 > \lambda_2 > \lambda_0$. We again first analyse the cartel's problem, which seeks to maximise the firms' aggregate discounted payoffs.

5.1 The Cartel's Problem

We can again restrict the cartel to Markov strategies $k^t = (k_1^t, k_2^t)$ with the posterior belief p_t as the state variable, where we write $k_i^t = 1(0)$ $(i = 1,2)$ if the cartel assigns firm i to method R (S). The value function of the cartel $v(p)$ satisfies

$$rv = \max_{(k_1,k_2)\in\{0,1\}^2} 2\lambda_0(1-v) + k_1\big\{\lambda_1 p[1-v-(1-p)v^{'}] - \lambda_0[1-v]\big\} \\ + k_2\big\{\lambda_2 p[1-v-(1-p)v^{'}] - \lambda_0[1-v]\big\} \quad (3)$$

The expression $2\lambda_0(1-v)$ is the expected flow payoff the cartel can guarantee itself by assigning both firms to method S. On the other hand, $\lambda_i p[1-v-v^{'}(1-p)] - \lambda_0(1-v)$ reflects the premium the cartel gets by assigning firm i to method R. By linearity, it would be without loss for the cartel to choose $\{k_i(p_t)\}_{t\geq 0}$ $(i = 1,2)$ with $k_i(p_t) \in \{0,1\}$, even if firms' efforts were divisible. The following proposition describes the cartel's solution. It shows that maximisation of joint payoffs requires the cartel to choose the myopically optimal method for firm 1, while assigning firm 2 to method S at some beliefs above its myopically optimal threshold. To state the theorem, we use the strictly decreasing and strictly convex functions $\mu(p) = (1-p)(\frac{1-p}{p})^{\frac{r}{\lambda_1+\lambda_2}}$ and $\mu_1(p) = (1-p)(\frac{1-p}{p})^{\frac{r+\lambda_0}{\lambda_1}}$.

Proposition 3 *The cartel's optimal solution is characterised by thresholds $p_1^* = \frac{\lambda_0}{\lambda_1}$ and $p_2^* \in (\frac{\lambda_0}{\lambda_2}, 1)$, such that, for $p \in (p_2^*, 1]$ ($p \in (0, p_1^*]$), both firms are assigned to method R (S). For $p \in (p_1^*, p_2^*]$, firm*

1 *is assigned to method R and firm 2 is assigned to method S. The cartel's value function is given by*

$$v(p) = \begin{cases} \frac{\lambda_1+\lambda_2}{r+\lambda_1+\lambda_2} p + C_{rr}\mu(p) & \equiv \check{v}_{rr}(p) \quad \text{if } p \in (p_2^*, 1], \\ \frac{\lambda_0}{r+\lambda_0} + \frac{r\lambda_1}{(r+\lambda_0)(r+\lambda_0+\lambda_1)} p + C_{rs}\mu_1(p) & \equiv \check{v}_{rs}(p) \quad \text{if } p \in (p_1^*, p_2^*], \\ \frac{2\lambda_0}{r+2\lambda_0} & \qquad\qquad \text{if } p \in [0, p_1^*]. \end{cases} \tag{4}$$

where $p_2^* \in (\frac{\lambda_0}{\lambda_2}, 1)$ *satisfies*

$$\check{v}_{rr}(p_2^*) = \check{v}_{rs}(p_2^*) = \frac{\lambda_0(\lambda_1+\lambda_2)}{r\lambda_2 + \lambda_0(\lambda_1+\lambda_2)}.$$

C_{rs} *and* C_{rr} *are constants of integration with* $C_{rs} = \frac{r\lambda_0(\lambda_1-\lambda_0)}{(r+\lambda_0)(r+2\lambda_0)(r+\lambda_0+\lambda_1)\mu_1(p_1^*)} > 0$*, and* $C_{rr} > 0$ *is determined from* $\check{v}_{rr}(p_2^*) = \check{v}_{rs}(p_2^*)$.

Proof. Proof is by a standard verification argument. Please refer to Appendix D for details, and Appendix A.2 for the ODEs satisfied by the cartel's value function. ■

The cartel's value function $v(p)$ is of class C^1, (strictly) increasing and (strictly) convex (on $(p_1^*, 1)$). At the optimum, firm 2 switches from method R to method S as soon as the belief drops below the threshold $p_2^* > \frac{\lambda_0}{\lambda_2}$. By contrast, if firm 2 were the only firm around, then it would have optimally switched to method S at the belief $\frac{\lambda_0}{\lambda_2}$. In the presence of firm 1, however, firm 2 optimally switches at a belief higher than its myopic threshold, while firm 1 optimally switches to method S at its myopic threshold $\frac{\lambda_0}{\lambda_1}$. Thus the cartel has firm 2 switch its action at a belief where the expected arrival rate on the innovative method is higher than that of the established method.

This a priori surprising information aversion by the cartel can intuitively be explained as follows. Since the game ends after the first breakthrough, there is no learning benefit from a breakthrough and hence, in the cartel's solution, no firm will be made to use method R for beliefs less than its myopic cutoff. This implies that firm 1 is the last firm to switch to method S at its myopic belief $p_1^* = \frac{\lambda_0}{\lambda_1}$. Since firm 1 is more productive than firm 2, the cartel would gain if it could contemporaneously substitute firm 1's experimentation for firm 2's. While such a contemporaneous substitution is not feasible, it is however indeed possible for the cartel to substitute *future* experimentation by firm 1 for *current* experimentation by firm 2. This intertemporal substitution, of course, comes at the price of delaying the expected time of the breakthrough. For any belief strictly greater than $\frac{\lambda_0}{\lambda_1}$, while more future experimentation by firm 1 leads to an expected positive gain, the cartel incurs an expected loss by giving up current experimentation by firm 2. At $\frac{\lambda_0}{\lambda_2}$, the myopic threshold belief of firm 2, this expected loss is equal to zero. This explains why the cutoff p_2^* is strictly greater than $\frac{\lambda_0}{\lambda_2}$. Formally this can be understood as follows. At any belief, the expected positive gain from making firm 2 use R is $(\lambda_2 p - \lambda_0)(1-v)$, and the expected loss from the environment becoming more pessimistic following hapless experimentation by firm 2 is $-\lambda_2 p(1-p)v'$. Since v is strictly convex and increasing in p

for $p \in (p_1^*, 1)$, we have $v^{'}(\frac{\lambda_0}{\lambda_2}) > 0$. This implies that, at $p = \frac{\lambda_0}{\lambda_2}$, the expected gain from firm 2 using R, $(\lambda_2 p - \lambda_0)(1-v) = 0$, is outweighed by the cost $-\lambda_2 p(1-p)v^{'} < 0$. The cartel's incentive to substitute future experimentation by the stronger firm for current experimentation by the weaker firm thus leads to a delay in the expected time of breakthrough, suggesting that collusion between asymmetric firms harms their research productivity by virtue of this *intertemporal substitution effect*.

5.2 Non-cooperative game

Our solution concept is Markov perfect equilibrium. Given k_j $(j = 1,2)$, if v_i $(i = 1,2; i \neq j)$ is the payoff of firm i in equilibrium, then we have

$$v_i = \max_{k_i \in \{0,1\}} \{\lambda_0(1-k_i)\alpha\, dt + \lambda_0(1-k_j)(1-\alpha)\, dt + k_i\lambda_i p\alpha\, dt + k_j\lambda_j p(1-\alpha)\, dt$$

$$+(1-r\, dt)[1-\lambda_0(1-k_i)\, dt - \lambda_0(1-k_j)\, dt - (k_i\lambda_i + k_j\lambda_j)p\, dt][v_i - (k_i\lambda_i + k_j\lambda_j)p(1-p)v_i^{'}dt]\}$$

$$\Rightarrow rv_i = \lambda_0[1-2v_i] + \max_{k_i \in \{0,1\}} k_i\{\lambda_i p[\alpha - v_i - (1-p)v_i^{'}] - \lambda_0[\alpha - v_i]\}$$
$$+k_j\{\lambda_j p[1-\alpha - v_i - (1-p)v_i^{'}] - \lambda_0[1-\alpha-v_i]\}. \tag{5}$$

Firm i $(i = 1,2)$ can guarantee itself an expected flow payoff of $\lambda_0(\alpha - v_i) + ((1-k_j)\lambda_0 + k_j\lambda_j p)(1-\alpha-v_i) = \lambda_0(1-2v_i) + k_j(\lambda_j p - \lambda_0)(1-\alpha-v_i)$ by using the established method (S). The term $\{\lambda_i p[\alpha - v_i - (1-p)v_i^{'}] - \lambda_0[\alpha - v_i]\}$ captures the premium firm i receives by using the innovative method. The expression $((1-k_j)\lambda_0 + k_j\lambda_j p)(1-\alpha-v_i)$ captures the payoff externality firm j exerts on firm i if it has a success, while $k_j\lambda_j p(1-p)v_i'$ captures the informational externality caused by firm j's hapless experimentation with the innovative method.

Best Responses:

Suppose $k_j = 0$ $(j \in \{1,2\})$ in an open neighbourhood of p. From (5) and ODE (25), we can see that using method R in a neighbourhood of p is optimal for firm i $(i \in \{1,2\}; i \neq j)$ if and only if

$$v_i \geq \frac{\lambda_0}{r + 2\lambda_0}$$

is satisfied in that neighbourhood.

Next, suppose $k_j = 1$ in an open neighbourhood of p. From (5) and ODE (29), we can infer that choosing R is optimal for firm i in a neighbourhood of p if and only if

$$v_i \geq \frac{\lambda_0\alpha[\lambda_1 + \lambda_2] + \lambda_1\lambda_2 p[1-2\alpha]}{r\lambda_i + \lambda_0(\lambda_1 + \lambda_2)} \tag{6}$$

is satisfied in that neighbourhood.

Our main result characterises the unique Markov perfect equilibrium of our game. For any level of patent protection α, both firms will use a cutoff strategy in equilibrium, that is, they use the innovative method if and only if the likelihood of it being good is above a threshold. Firm 1 uses the innovative method (R) in the belief region $(\frac{\lambda_0}{\lambda_1}, 1]$, and the established method (S) otherwise. Firm 2 uses the innovative method (R) on $(\hat{p}_2(\alpha), 1]$ and the established method (S) otherwise. Thus, while firm 1's cutoff is independent of α, firm 2's equilibrium threshold is a decreasing function of α; the stronger the level of patent protection α, the more firm 2 will be inclined to use the innovative method. Indeed, the theorem shows that *firm 2 will use the innovative method too much*, compared to the cartel solution, as soon as there is *some* level of patent protection, i.e., whenever $\alpha > \frac{1}{2}$. If patent protection is relatively weak, i.e., $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$, firm 2 will use the innovative method less than if it were by itself; for $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, by contrast, it uses the innovative method beyond the myopically optimal threshold. If and only if $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$ will it behave myopically, thereby minimising the expected time to a first breakthrough.

This can be intuitively understood as follows. Firms have two goals: (1) on account of discounting, they want the breakthrough to occur as soon as possible; (2) on account of the payoff rivalry between them, they both want to be the one achieving the breakthrough. The level of patent protection determines the relative importance of these goals in the firms' objectives. When there is no patent protection, i.e., $\alpha = \frac{1}{2}$, the payoff rivalry is shut down and firms behave cooperatively in the unique Markov perfect equilibrium of the non-cooperative game; i.e., $\hat{p}_2(\frac{1}{2}) = p_2^*$. As soon as $\alpha > \frac{1}{2}$, some payoff rivalry comes into play, as both firms want to be the first inventor achieving the breakthrough; as a result, $\hat{p}_2(\alpha) < p_2^*$ for all $\alpha > \frac{1}{2}$. At the belief $p = \frac{\lambda_0}{\lambda_2}$, the individual myopic expected payoff to firm 2 is the same for both methods. However, by using method R, firm 2 is producing additional information, implying that, if there is no breakthrough, firms become more pessimistic about the innovative method. In equilibrium, though, firm 1 uses method R until the belief reaches $p_1^* = \frac{\lambda_0}{\lambda_1}$. Thus, as the belief decreases due to firm 2's unsuccessful use of method R, the time firm 1 spends using R is reduced. Based on the current belief $p = \frac{\lambda_0}{\lambda_2}$, this reduces both the overall chance of a breakthrough on the innovative method and, more particularly, the chances of a breakthrough by firm 1. While the former is bad news for firm 2, the latter is good news (provided $\alpha > \frac{1}{2}$); now, how firm 2 trades-off these two countervailing effects depends on the level of patent protection α. For $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$, the two effects just cancel out and firm 2 best-responds by behaving myopically, i.e., as though it were by itself. For high levels of patent protection ($\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$), the desire to be first dominates at the myopic threshold and firm 2 extends the use of the innovative method below its myopic threshold, while for low levels of patent protection ($\alpha \in (\frac{1}{2}, \frac{r+\lambda_0}{r+2\lambda_0})$), firm 2's cooperative motive prevails at the myopic threshold, and its equilibrium cutoff satisfies $\hat{p}_2(\alpha) \in (\frac{\lambda_0}{\lambda_2}, p_2^*)$.

Thus, as $\hat{p}_2(\alpha) \leq p_2^*$, the (negative) payoff externality overwhelms the (positive) informational externality, and firms experiment (strictly) too much in equilibrium (if $\alpha > \frac{1}{2}$ and $\lambda_1 > \lambda_2$). If the firms are perfectly symmetric, i.e., $\lambda_1 = \lambda_2$, the cartel and the non-cooperative firm no longer have any divergent incentives to swap current experimentation by one firm for future experimentation by the other firm, and the friction at the heart of the model, and therefore the *intertemporal substitution effect* disappear. Indeed, as we have seen in Section 4, when firms are symmetric, the unique equilibrium coincides with the cartel solution and minimises the expected waiting time for a breakthrough. When $\alpha = \frac{1}{2}$, there is no longer any rivalry between the firms, which consequently approach intertemporal substitution exactly as a cartel would, swapping current experimentation by the weaker firm for future experimentation by the stronger firm, so that equilibrium behaviour will coincide with the cartel's behaviour.

Theorem 1 *There exists a unique Markov perfect equilibrium. Equilibrium strategies are given by* $k_1^{-1}(1) = (p_1^*, 1]$ *and* $k_2^{-1}(1) = (\hat{p}_2(\alpha), 1]$. *Firm 2's cutoff* $\hat{p}_2(\alpha)$ *is a strictly decreasing, continuously differentiable, function, satisfying* $\hat{p}_2(\frac{1}{2}) = p_2^*$, $\hat{p}_2(\frac{r+\lambda_0}{r+2\lambda_0}) = \frac{\lambda_0}{\lambda_2}$, *and* $\hat{p}_2(1) > p_1^*$.

The firms' equilibrium payoffs are given by

$$v_1(p) = \begin{cases} \frac{\lambda_1\alpha+\lambda_2(1-\alpha)}{r+\lambda_1+\lambda_2} p + C_1^{rr}\mu(p) & \equiv v_1^{rr}(p) \quad \text{if } p \in (\hat{p}_2(\alpha), 1] \\ \frac{\lambda_0(1-\alpha)}{r+\lambda_0} + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\left[\alpha - \frac{\lambda_0(1-\alpha)}{r+\lambda_0}\right] + C_1^{rs}\mu_1(p) & \equiv v_1^{rs}(p) \quad \text{if } p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2(\alpha)] \\ \frac{\lambda_0}{r+2\lambda_0} & \quad \text{if } p \in (0, \frac{\lambda_0}{\lambda_1}], \end{cases} \tag{7}$$

and

$$v_2(p) = \begin{cases} \frac{\lambda_2\alpha+\lambda_1(1-\alpha)}{r+\lambda_1+\lambda_2} p + C_2^{rr}\mu(p) & \equiv v_2^{rr}(p) \quad \text{if } p \in (\hat{p}_2(\alpha), 1], \\ \frac{\lambda_0\alpha}{r+\lambda_0} + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\left[1 - \alpha - \frac{\lambda_0\alpha}{r+\lambda_0}\right] + C_2^{rs}\mu_1(p) & \equiv v_2^{rs}(p) \quad \text{if } p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2(\alpha)] \\ \frac{\lambda_0}{r+2\lambda_0} & \quad \text{if } p \in (0, \frac{\lambda_0}{\lambda_1}], \end{cases} \tag{8}$$

respectively.

The threshold $\hat{p}_2(\alpha)$ *is implicitly defined by* $v_2^{rs}(\hat{p}_2(\alpha)) = \frac{\lambda_0\alpha[\lambda_1+\lambda_2]+\lambda_1\lambda_2\hat{p}_2(\alpha)[1-2\alpha]}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. *The constants of integration are determined by value matching, i.e.,* $C_1^{rs} > 0$ *is given by* $v_1^{rs}(p_1^*) = \frac{\lambda_0}{r+2\lambda_0}$ *and* C_2^{rs} *by* $v_2^{rs}(p_1^*) = \frac{\lambda_0}{r+2\lambda_0}$. *We have* $C_2^{rs} > 0$ ($C_2^{rs} < 0$) *if and only if* $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$ ($\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$), *and* $C_2^{rs} = 0$ *if and only if* $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$. *Similarly, the constants of integration* C_1^{rr} *and* $C_2^{rr} > 0$ *are determined by* $v_1^{rr}(\hat{p}_2(\alpha)) = v_1^{rs}(\hat{p}_2(\alpha))$, *and* $v_2^{rr}(\hat{p}_2(\alpha)) = v_2^{rs}(\hat{p}_2(\alpha))$, *respectively. The function* v_2 *is smooth, while* v_1 *is smooth everywhere except at* $p = \hat{p}_2(\alpha)$.

Proof. Existence of the equilibrium follows from standard verification arguments, while uniqueness follows from the Bellman equation (5) and the relevant ODEs (Appendix A.2). Please see Appendix E for a detailed proof. ■

For high values of patent protection ($\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$), firm 2's value function is decreasing and concave in the region where only firm 1 uses method R; it is convex in the range where both firms use it. It has an inflection point at $\hat{p}_2(\alpha)$, where firm 2 switches methods, and eventually becomes increasing as firms become very optimistic about method R. For low levels of patent protection ($\alpha \leq \frac{r+\lambda_0}{r+2\lambda_0}$), by contrast, v_2 is increasing and convex throughout.

Our analysis would suggest that, in the knife-edge case of perfectly symmetric firms, both the cartel and the non-cooperative firms would behave consistently with the goal of the social planner who wants to speed up the expected time of breakthrough as much as possible. However, as soon as one firm is better capable of handling the innovative method, the cartel steers the less productive firm away from the innovative method too soon, i.e., there is *insufficient clustering* in the innovative method. What happens in non-cooperative equilibrium in the asymmetric case depends on the level of patent protection, as summarised in the following corollary.

Corollary 1 *Suppose the firms are asymmetric, i.e., $\lambda_1 > \lambda_2$. The expected time to the first breakthrough is minimised for the level of patent protection $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$. If patent protection is strong, i.e., $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, this expected time is delayed on account of* excessive clustering *of innovative efforts. If patent protection is weak, i.e., $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$, this expected time is delayed on account of* insufficient clustering *of innovative efforts. The delay due to insufficient clustering is worst when there is no patent protection at all (i.e. $\alpha = \frac{1}{2}$) or firms form a cartel.*

Thus, our analysis would suggest that, besides watching out for collusion between firms, policymakers who endeavour to speed up the expected time of a decisive breakthrough should be wary of both too strong patent regimes as well as too weak ones. Indeed, the former will tend to exacerbate firms' rivalry to the point where the race to be first makes them engage in excessive clustering of innovative efforts. The latter, by contrast, makes firms behave "too cooperatively" in the sense that the weaker firm will be too inclined to substitute *future* experimentation by its partner for its own *current* experimentation, leading to insufficient clustering of research efforts. Our findings thus suggest a formal channel explaining the puzzling decrease in R&D productivity connected with a strengthening of patent protections, which has been noted in the empirical literature.

Consequently, a social planner who intends to achieve the shortest expected time to the first discovery should set patent protection to $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$. It is interesting to observe that this optimal patent protection is a function of λ_0 and r only, and, in particular, does not depend on the values of λ_1 and λ_2. This a-priori surprising feature can be intuitively explained as follows. Recall that the optimal patent protection has the property that, given firm 1's equilibrium action, firm 2 finds it optimal to switch to method S at the belief $\frac{\lambda_0}{\lambda_2}$. Given that, in equilibrium, firm 1 will play R in a neighborhood of $\frac{\lambda_0}{\lambda_2}$ regardless of firm 2's action, firm 2's local free-riding payoff is independent of its own action. Hence the optimal α will not depend on firm 1's breakthrough rate λ_1. Furthermore, at the belief $\frac{\lambda_0}{\lambda_2}$, the

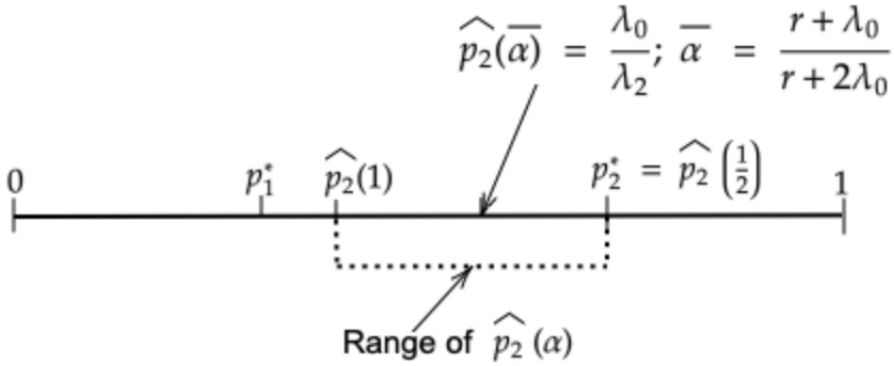

Figure 2: Cartel, equilibrium and social optimum

effective breakthrough rate for firm 2 is λ_0, irrespectively of the research method it chooses. Hence, the optimal α will not depend on λ_2.

One general criticism against higher levels of patent protection is that it can adversely affect the consumers' welfare by making the market more concentrated. Since that channel is absent in our framework as S is independent of α, one a-priori might expect that a higher level of patent protection is desirable to enhance research productivity. Our analysis rather surprisingly suggests that even when consumers are not directly affected by the level of patent protection, excessive patent protection is not desirable as it slows down the time to discovery. The comparison between the cartel solution, non-cooperative solution and the social benchmark is depicted in Figure 2.

6 Extensions

In this section, we consider a more general version of our baseline model discussed above. Specifically, we shall explicitly model the impact of the patent regime on the post-innovation product market (Subsection 6.1), incorporate explicit costs of conducting research (Subsections 6.2 and 6.3), as well as a higher payoff from a breakthrough that is achieved with the innovative method (Subsection 6.4). We explore how each of these modifications affects the results obtained in our main model.

We assume that, after the process innovation takes place, the demand for the product is described by the inverse demand function $P(Q) = \tilde{\Pi}(A - BQ)$, where $\tilde{\Pi} = 1$ if the breakthrough has been achieved by the established method and $\tilde{\Pi} = \Pi \geq 1$ otherwise. We normalize production costs after an invention to 0. Furthermore, we assume that patent protection gives the winner monopoly power over a share $2\alpha - 1$ of the product market; in the remaining share $2(1-\alpha)$, the two firms are engaged in Cournot competition. It follows that the winner gets $\frac{A^2\tilde{\Pi}(10\alpha-1)}{36B} \in \left[\frac{A^2\tilde{\Pi}}{9B}, \frac{A^2\tilde{\Pi}}{4B}\right]$, while the loser gets $\frac{2A^2\tilde{\Pi}(1-\alpha)}{9B} \in \left[0, \frac{A^2\tilde{\Pi}}{9B}\right]$.

In this section, we denote the established method as arm 0 and the innovative method R as arm 1. If the flow cost of conducting research is $s > 0$, then in the non-cooperative game, the Bellman equation of player i is given by

$$rv_i(p) = k_0^{(j)}\beta_0^{(i)}(p,v_i) + k_1^{(j)}\beta_1^{(i)}(p,v_i) + \max_{(k_0^{(i)},k_1^{(i)})}\left\{k_0^{(i)}[b_0^{(i)}(p,v_i) - s] + k_1^{(i)}[b_1^{(i)}(p,v_i) - s]\right\}, \tag{9}$$

where

$$\beta_0^{(i)}(p,v) := \lambda_0\left(\frac{2A^2(1-\alpha)}{9B} - v(p)\right),$$

$$\beta_1^{(i)}(p,v) := p\lambda_j\left(\frac{2A^2(1-\alpha)}{9B}\Pi - v(p) - (1-p)v'(p)\right),$$

$$b_0^{(i)}(p,v) := \lambda_0\left(\frac{A^2(10\alpha-1)}{36B} - v(p)\right),$$

$$b_1^{(i)}(p,v) := p\lambda_i\left(\frac{A^2(10\alpha-1)}{36B}\Pi - v(p) - (1-p)v'(p)\right).$$

Please refer to Appendix F for the closed form expressions of the payoffs of player i under each possible action profile, and also the best response analysis for each possible action of player j.

6.1 The Product Market

We will first explore the effect of explicitly modelling the product market after the process innovation. In order to do this, we assume $\Pi = 1$ and $s = 0$ in this subsection. As we have outlined above, post innovation the aggregate profit is given by $\frac{A^2}{36B}(7+2\alpha)$. Thus, as patent strength, and hence market power, increases, industry profit strictly increases.

The following proposition concludes that the conclusions obtained in our main analysis are robust to this extension. To state the proposition, we define $\bar{\alpha} = \frac{8r+9\lambda_0}{8r+18\lambda_0} \in \left(\frac{1}{2}, 1\right)$.

Proposition 4 *There exists a threshold p_2 satisfying $0 < \frac{\lambda_0}{\lambda_1} < p_2 < 1$ such that, on $\left(0, \frac{\lambda_0}{\lambda_1}\right]$, both firms use arm 0, on $\left(\frac{\lambda_0}{\lambda_1}, p_2\right]$, firm 1 uses arm 1 and firm 2 uses arm 0. Finally, for all $p \in (p_2, 1]$, both firms use arm 1. The threshold p_2 is a strictly decreasing and continuously differentiable function of α satisfying $p_2 < \frac{\lambda_0}{\lambda_2}$ if and only if $\alpha > \bar{\alpha}$, $p_2 > \frac{\lambda_0}{\lambda_2}$ if and only of $\alpha < \bar{\alpha}$, and $p_2 = \frac{\lambda_0}{\lambda_2}$ if and only if*

$\alpha = \bar{\alpha}$.

The proposition is a special case of Proposition 5, which is proved in Appendix G.

Thus, taking the post-innovation product market into account in a manner such that industry profits depend on the patent strength α, does not disturb any of the qualitative results of our baseline model. As in our baseline model, the optimal patent strength depends only on the discount rate r and the breakthrough rate λ_0 with the established method—the intuitive explanation is the same as in the preceding section.

We conclude this subsection by discussing how the level of patent protection affects social welfare W defined in 1. In the current extension, the post-innovation social surplus $TS = \frac{A^2(7+2\alpha)(5-2\alpha)}{72B}$. Since TS is strictly decreasing in α, for $\alpha \in [\frac{8r+9\lambda_0}{8r+18\lambda_0}, 1]$, W is strictly decreasing in α. For $\alpha \in [\frac{1}{2}, \frac{8r+9\lambda_0}{8r+18\lambda_0})$, the effect of α on W is ambiguous, as, while a reduction in α increases the expected time to discovery, it also increases TS. Hence, the welfare optimum is achieved at an $\alpha \in (\frac{1}{2}, \frac{8r+9\lambda_0}{8r+18\lambda_0}]$. We illustrate this point in Figure 3 by means of a numerical example[9]. Figure 3 shows that W reaches its maximum at a $\tilde{\alpha} \in (\frac{1}{2}, \bar{\alpha})$, illustrating our conclusion that even when post-innovation surplus is adversely affected by a higher level of patent protection, some level of patent protection might be desirable.

6.2 Incorporating Cost of research

In this subsection, we will incorporate a flow cost of conducting research, which is given by $s > 0$. In the absence of explicit research costs, players will always conduct research, using either the established (S), or the innovative (R), approach. If the cost of conducting research is positive, however, it is possible that one or both the research methods are dominated. In the current section, we will consider only values of $s > 0$ such that, in equilibrium, at least one player uses R for some beliefs. Further, we continue to explicitly model the product market, while assuming $\Pi = 1$. For a given level of patent protection, the analysis is non-trivial only when $\frac{s}{\lambda_1} < \frac{A^2}{36B}(10\alpha - 1)$; otherwise, method R would be too expensive for both firms even if it was known to be good. As we assume $\lambda_1 > \lambda_2 > \lambda_0$, S would be dominated as well in this case, and hence no research would take place. Since $\alpha \leq 1$, we shall therefore assume throughout that $\frac{s}{\lambda_1} < \frac{A^2}{4B}$. This ensures that for the more productive firm, method R is not dominated for some values of α.

We will first consider the situation when the method S is not dominated for some values of $\alpha \in (\frac{1}{2}, 1]$. This implies $\frac{s}{\lambda_0} < \frac{A^2}{4B}$. The following proposition shows that, if the cost of conducting research is lower than a certain threshold, then there exists a cutoff equilibrium, as in our baseline model. To state the following proposition, we define $\bar{\alpha}_s = \frac{8r+9\lambda_0}{8r+18\lambda_0} + \frac{s}{8r+18\lambda_0}\frac{36B}{A^2}$.

[9] Parameter values are $\lambda_0 = 18$, $\lambda_1 = 30$, $\lambda_2 = 22$, and $r = 2$.

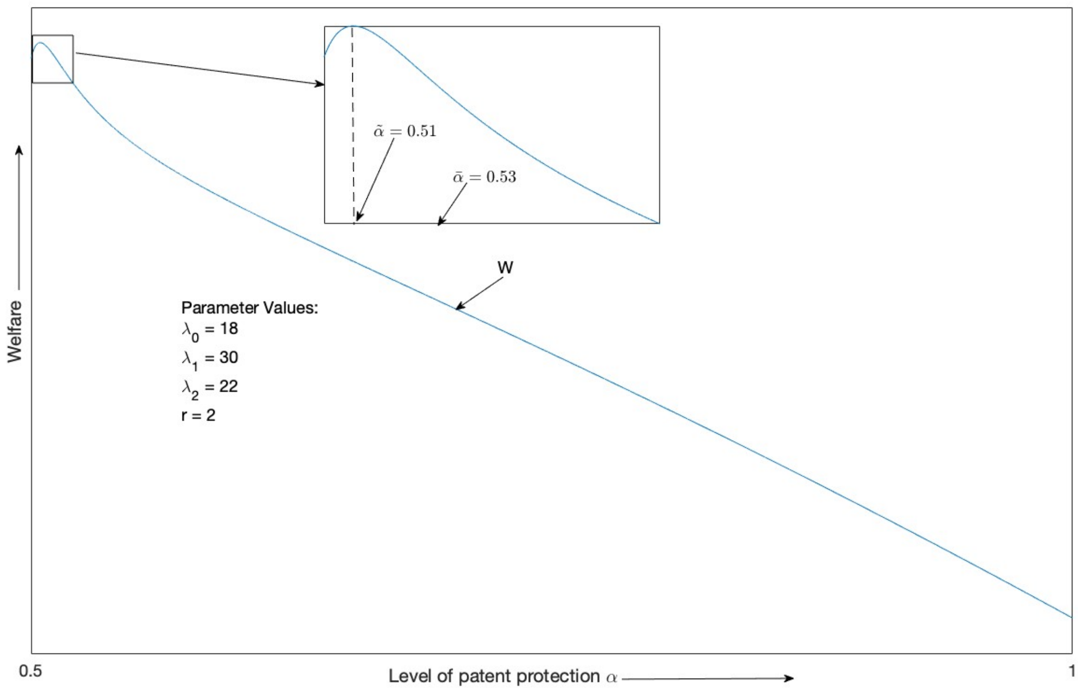

Figure 3: W for each level of α

Proposition 5 *Suppose $\frac{s}{\lambda_0} < \frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$. Then, there exists a cutoff equilibrium in which, on $(0, \frac{\lambda_0}{\lambda_1}]$, both firms use arm 0, on $(\frac{\lambda_0}{\lambda_1}, p_2]$, firm 1 uses arm 1 and firm 2 uses arm 0, and for all $p \in (p_2, 1]$, both firms use arm 1, for some threshold p_2 satisfying $0 < \frac{\lambda_0}{\lambda_1} < p_2 < 1$. This threshold p_2 is a strictly decreasing and continuously differentiable function of α, satisfying $p_2 < \frac{\lambda_0}{\lambda_2}$ if and only if $\alpha > \bar{\alpha}_s$, $p_2 > \frac{\lambda_0}{\lambda_2}$ if and only if $\alpha < \bar{\alpha}_s$, and $p_2 = \frac{\lambda_0}{\lambda_2}$ if and only if $\alpha = \bar{\alpha}_s$.*

The proof of the proposition is relegated to Appendix G. The proposition shows that our previous results are qualitatively robust to the introduction of small research costs. The optimal patent strength is now a function of λ_0, r and s; it is still independent of parameters λ_1 and λ_2, for the same reasons as in our main analysis. However, once the research cost s exceeds $\lambda_0\frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$, incentives regarding the choice of research method change, as the following corollary shows.

Corollary 2 *If $\frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r} < \frac{s}{\lambda_0} < \frac{A^2}{4B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$, the cutoff equilibrium of our main analysis exists if and only if the level of patent protection α exceeds the threshold $\alpha^{*s} = \frac{1}{10} + \frac{s}{10\lambda_0}\frac{36B}{A^2}\frac{\lambda_0\lambda_1+\lambda_2 r}{\lambda_2 r}$.*

If $\frac{s}{\lambda_0} < \frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$, method S dominates giving up, for all levels of patent protection α. Thus, the exact value of the research cost s has no impact on the players' incentives to free-ride, as both research methods entail the same costs s. If $\frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r} < \frac{s}{\lambda_0} < \frac{A^2}{4B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$, however, this continues to be true if and only if $\alpha > \alpha^{*s}$; for levels of patent protection below this threshold, firms prefer to give up, rather than use method S, so that our equilibrium from Theorem 1 does not survive. Thus, for higher research costs s, some post-innovation monopoly power becomes necessary to induce both firms to use both research methods in equilibrium.

Next, we will focus our attention on the values of s such that the use of the established research method S is strictly dominated for all levels of patent protection, i.e., $\frac{s}{\lambda_0} > \frac{A^2}{4B}$. Thus, relevant actions for players are either to conduct research using the innovative method R or to conduct no research. This implies firms' incentives to free-ride are related to the magnitude of the cost of conducting research. In the following proposition, we first determine a condition that ensures the existence of an equilibrium in cutoff strategies.

Proposition 6 *Suppose* $\lambda_0 \frac{A^2}{4B} < s < \lambda_2 \frac{A^2}{4B}$. *There exists an* $\bar{\alpha}(\lambda_1, \lambda_2) \in (\frac{1}{2}, 1)$ *such that, for* $\alpha \in (\bar{\alpha}(s, \lambda_1, \lambda_2), 1]$*, an equilibrium in cutoff strategies exists.*

The proof of the Proposition is relegated to Appendix H.

The above proposition shows that when the method S is strictly dominated, an equilibrium in cutoff strategies exists for high enough levels of patent protection. In this cutoff equilibrium, there exists a range of beliefs in which both firms use the method R. This suggests that, to incentivise both firms to use R, the value of α should be high, lest any firm have an incentive to deviate and free-ride on the other's research. The following proposition formalises this intuition.

Proposition 7 *Suppose* $\max\{\lambda_0 \frac{A^2}{4B}, \frac{r\lambda_2}{r+\lambda_1} \frac{A^2}{9B}\} < s < \lambda_2 \frac{A^2}{4B}$. *If there exists an equilibrium in which both firms use R for some range of beliefs, then α exceeds some threshold strictly larger than* $\frac{1}{2}$.

Please refer to Appendix I for the proof. Thus, if research is so costly that the established method S is dominated, high levels of patent protection are necessary for both firms to conduct research in equilibrium. Hence, in such a scenario, a policy maker who values a fast discovery should set α high, i.e., allow for more monopoly power to the discoverer in the post-innovation product market.

6.3 Fixed Cost of Market Entry

In our baseline model, we assume that firms do not have to pay a participation cost in order to engage in R& D activities. In this subsection, we introduce such a fixed cost of entry for the weaker firm, assuming that the strong firm has already entered the market. In the following proposition, we show that for every p, v_2 is decreasing in α, implying that the range of fixed costs for which the weaker firm is willing to enter monotonically decreases in the strength of patent protection.

Proposition 8 *In the unique equilibrium of our baseline model,* $v_2|_{\left[\frac{\lambda_0}{\lambda_1}, 1\right]}$ *is decreasing in* α.

Please refer to Appendix J for the proof.

The above proposition implies that, for any given p, it is more likely for the participation constraint to be violated as the level of patent protection α increases, thus inducing firm 2 to quit. From our baseline analysis, we already know that, when α exceeds $\frac{r+\lambda_0}{r+2\lambda_0}$, discovery is delayed in expectation. This means that, in the presence of fixed costs of entry, there can be further delays to discovery for higher patent protections $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, as firm 2 might drop out. Firm 1, for its part, will still use the threshold $\frac{\lambda_0}{\lambda_1}$, regardless of whether firm 2 enters the market.

6.4 Explicit Modelling of $\Pi > 1$

In this subsection we will explore the effect of having a higher payoff in the event the breakthrough has been achieved through the innovative method R, i.e., the case of $\Pi > 1$. While we retain the product market in this subsection, we assume $s = 0$. From the Bellman equation 9), we now have the following:

When both players use method S, then each gets the payoff $v^{ss} = \frac{\lambda_0}{r+2\lambda_0}\frac{A^2(7+2\alpha)}{36B}$. When player j uses the method R, it is optimal for player i to use R as long as his payoff v_i satisfies

$$v_i \geq \psi_i = \frac{A^2}{36B}\left[\frac{\lambda_0(\lambda_1+\lambda_2)(10\alpha-1)-\lambda_1\lambda_2\Pi[18\alpha-9]p}{r\lambda_i+\lambda_0(\lambda_1+\lambda_2)}\right]$$

If player i uses R and player j uses S, then player i's payoff v_i^{rs} and player j's payoff v_j^{rs} are given by

$$v_i^{rs} = \frac{\lambda_0}{r+\lambda_0}\frac{2A^2(1-\alpha)}{9B}\left(1-\frac{p\lambda_1}{r+\lambda_0+\lambda_1}\right)+\frac{\lambda_1\Pi p}{r+\lambda_0+\lambda_1}\frac{A^2(10\alpha-1)}{36B}+C_i^{rs}\mu_1(p) \tag{10}$$

$$v_j^{rs} = \frac{\lambda_0}{r+\lambda_0}\frac{A^2(10\alpha-1)}{36B}\left(1-\frac{p\lambda_1}{r+\lambda_0+\lambda_1}\right)+\frac{\lambda_1\Pi p}{r+\lambda_0+\lambda_1}\frac{2A^2(1-\alpha)}{9B}+C_j^{rs}\mu_1(p) \tag{11}$$

where $\mu_1(p) = (1-p)[\frac{1-p}{p}]^{\frac{r+\lambda_0}{\lambda_1}}$. C_i^{rs} and C_j^{rs} are constants of integration, which are determined from the relevant boundary conditions.

In the current subsection, in addition to the case $\lambda_1 > \lambda_0$, which is the setting we adopt throughout the paper, we also consider the variation where $0 < \lambda_2 \leq \lambda_1 < \lambda_0$ but $0 < \lambda_0 < \lambda_2\Pi \leq \lambda_1\Pi$. This reflects a scenario where it is more difficult to get a breakthrough using a good innovative method R but the payoff from the breakthrough is very high.

We first show that, for low levels of firm asymmetry, there does not exist any equilibrium in cutoff strategies as in the main model, as the following proposition shows.

Proposition 9 *Consider* $\Pi > 1$ *and* $\alpha \in (\frac{1}{2}, 1]$.

(i) If $\lambda_0 < \lambda_1$, then there exists a $\bar{\lambda}_2 \in (\lambda_0, \lambda_1)$ such that, if $\lambda_2 \in (\bar{\lambda}_2, \lambda_1)$, there does not exist an equilibrium in cutoff strategies as in the main model.

(ii) If $\lambda_0 > \lambda_1$,then there exists a $\bar{\lambda}_2 \in (0, \lambda_1)$ such that, if $\lambda_2 \in (\bar{\lambda}_2, \lambda_1)$, there does not exist an equilibrium in cutoff strategies as in the main model.

Proof of this Proposition is relegated to Appendix K.

The reason for the non-existence of an equilibrium in cutoff strategies is v_2^{rs} having a strictly positive slope at $\underline{p}$ if $\Pi > 1$. Verbally, this means that, for a higher payoff from the innovative method ($\Pi > 1$), in some right-neighborhood of $\underline{p}$, the firm using S gets a benefit. This is the case because $\underline{p} < \frac{\lambda_0}{\lambda_1}$. Indeed, at the belief $\frac{\lambda_0}{\lambda_1}$, the breakthrough rates are the same for both methods. From our previous analysis, we have seen that, for $\Pi = 1$, no player ever uses R below the belief $\frac{\lambda_0}{\lambda_1}$. However, when $\Pi > 1$, in the conjectured equilibrium, the last player to use R does so all the way down to the belief $\underline{p} < \frac{\lambda_0}{\lambda_1}$. Hence, there exists a range of beliefs at which firm 1 is using R when the instantaneous breakthrough rate is higher on S. As long as there is some competition (i.e. $\alpha > \frac{1}{2}$), this is beneficial to firm 2. Note that for extreme competition ($\alpha = 1$) firm 2 only cares about the breakthrough rate of the opponent. This explains the result.

In Figure 4, we illustrate in an example[10] that, if λ_2 is very close to λ_1, an equilibrium in cutoff strategies will not exist. As one can observe, this happens because in any such conjectured equilibrium, using method R becomes dominant for firm 2 at a lower belief than for firm 1, i.e., $\hat{p}_2 < p_1^{'}$.

Conversely, an equilibrium in cutoff strategies will reappear if the parameters are such that using method R becomes dominant for firm 1 at a belief that is strictly lower than the corresponding belief for firm 2. The following Proposition establishes a sufficient condition for the existence of the equilibrium in cutoff strategies.

Proposition 10 *Suppose $\Pi > 1$. For any $\alpha \in [\frac{1}{2}, 1)$, there exists a $\bar{r}(\alpha)$ such that, for all $r > \bar{r}(\alpha)$, an equilibrium in cutoff strategies exists when agents are very asymmetric (when λ_1 is large or λ_2 is very small). The threshold $\bar{r}(\cdot)$ is continuous, strictly increasing and satisfies $\bar{r}(\frac{1}{2}) = 0$ and $\lim_{\alpha \uparrow 1} \bar{r}(\alpha) = \infty$.*

The proof is relegated to Appendix L

The preceding proposition suggests the following. Suppose that $r > \bar{r}(\alpha)$ for given values of α and λ_0. This is indeed the case for the example[11] in Figure 4. Then, by making firms more asymmetric, we can guarantee the existence of an equilibrium in cutoff strategies. This is illustrated in Figure 5.[12] As we can see, $p_1^{'} < \hat{p}_2$ in this case, which ensures that the conjectured equilibrium is indeed an equilibrium.

[10]Parameter values for this figure: $\lambda_0 = 1$; $\lambda_1 = 1.14$; $\lambda_2 = 1.1$; $\Pi = 1.5$; $r = 1$ and $\alpha = 0.6$.

[11]$\lambda_0 = 1$ and $\alpha = 0.6$ lead to $\bar{r}(\alpha) = 0.56$.

[12]Parameter values for this figure: $\lambda_0 = 1$; $\lambda_1 = 1.5$; $\lambda_2 = 1.1$; $\Pi = 1.5$; $r = 1$ and $\alpha = 0.6$.

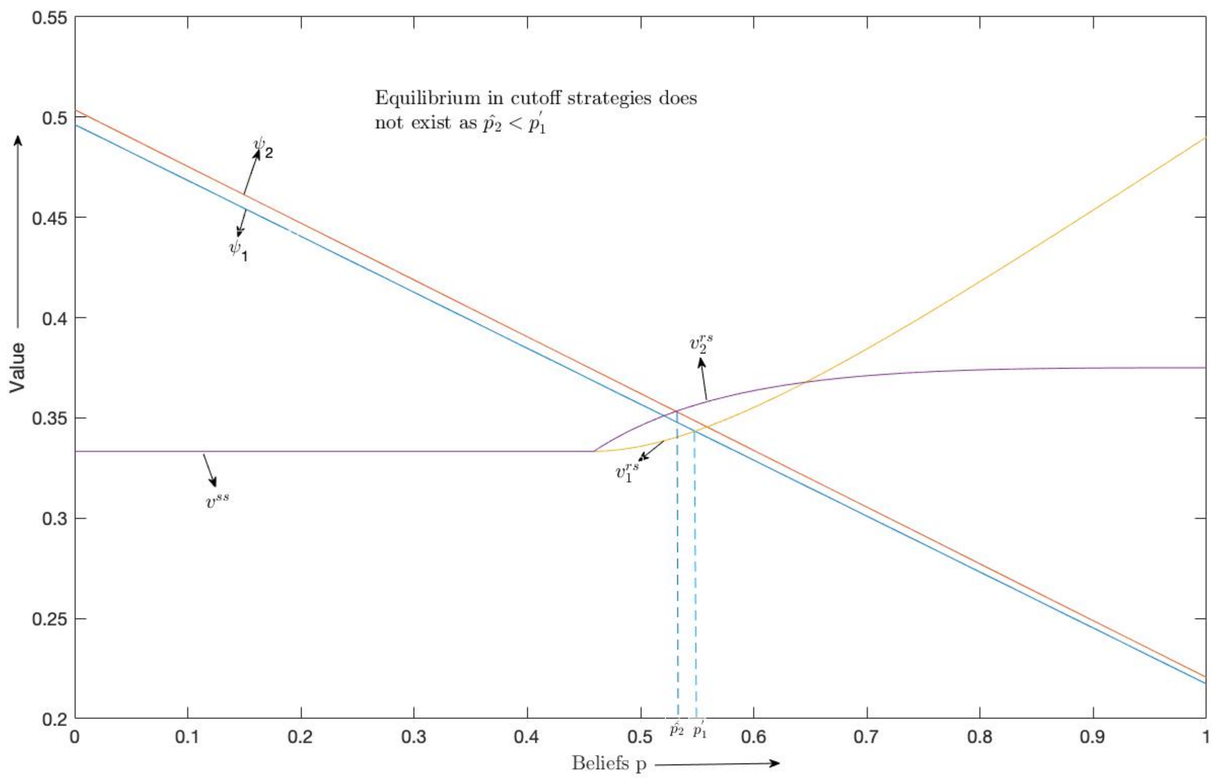

Figure 4: No equilibrium in cutoff strategies for a low degree of asymmetry

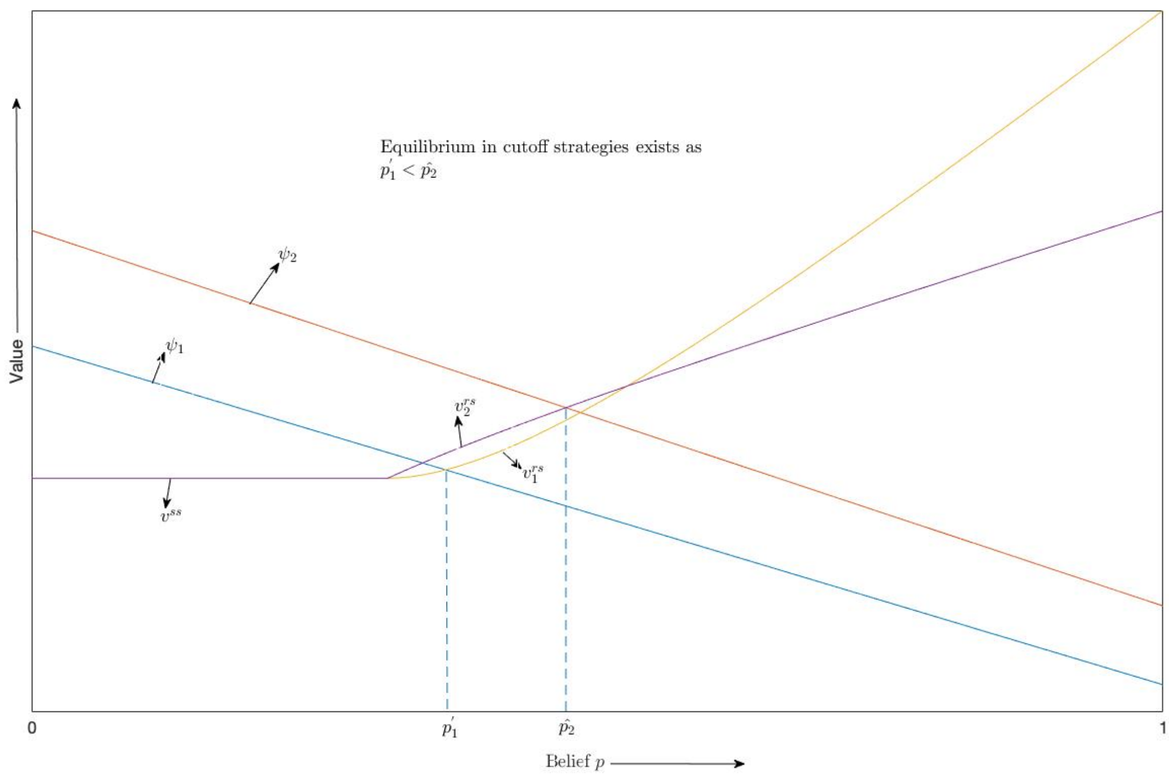

Figure 5: Equilibrium in cutoff strategies

7 Conclusion

We have shown that, in a patent-race model with dynamic learning and optimal readjustment of project selection, the combination of payoff externalities and asymmetric players gives rise to higher amounts of experimentation in equilibrium than in the cartel's solution. This effect is the stronger the more potent the regime of patent protection. The equilibrium expected time to breakthrough is minimised for an interior level of patent protection. This expected time to breakthrough is delayed on account of excessive (insufficient) clustering of experimentation if patent protection is above (below) this threshold.

From a policy point of view, our main contribution lies in the identification of the intertemporal-substitution effect. Our baseline model is the simplest setting we can think of that features this effect; in fact, it is the *only* source of friction in our baseline model. This is illustrated by the fact that efficiency obtains in both the cartel problem and in equilibrium when firms are symmetric, see Propositions 1 and 2. Indeed, for the intertemporal-substitution effect to arise, it is necessary that there be a strictly weaker and a strictly stronger firm. Indeed, by virtue of this effect, the weaker firm distorts its choice of method, as it is willing to incur a reduction in its success rate by over-(under-) using the innovative option in the adversarial (cooperative) environments induced by strong (weak) patent protection.

Interestingly, the threshold patent protection, at which the effect changes sign, does not depend on the properties of the innovative option (i.e., on λ_1 and λ_2). When we introduce a positive cost of conducting research $s > 0$, the usual trade-off re-appears as well in our model, in that stronger patent protection, and therefore greater monopoly power and lower welfare in the post-innovation product market, become necessary to incentivise exploration by firms. Whatever the research cost, however, policymakers ought to be aware that the choice of patent regime impacts firms' strategic choice of research avenue, and thereby the expected arrival of an innovation, through the intertemporal-substitution effect. This effect arises whenever *asymmetric* firms are capacity-constrained and their research efforts impact their competitors' beliefs about an innovative technology.

In contrast to models of purely informational externalities, such as Keller, Rady and Cripps (2005), our Markov perfect equilibrium is unique. It is furthermore in cutoff strategies, while there does not exist an equilibrium in cutoff strategies in Keller, Rady and Cripps (2005). In our setting, moreover, equilibrium deviates from the cooperative solution because of higher information production, while all equilibria in Keller, Rady and Cripps (2005) have the feature that players experiment too little compared to the cooperative benchmark.

We have confined our analysis to a two-player setting. While the analysis becomes increasingly complex as the number of players grows, we should expect our main qualitative insights to carry over to a setting with $N \geq 3$ players. In particular, the intertemporal-substitution effect, by virtue of which the adversarial (cooperative) setting induced by strong (weak) patent protection provides weaker firms with excessive (insufficient) incentives to use innovative research avenues, should carry over to a setting with an arbitrary number of firms. We should furthermore expect the insight that greater asymmetry among firms favours the existence of cutoff equilibria[13] to apply in a setting with N players as well, as a cutoff equilibrium exists if and only if the innovative method becomes dominant for the stronger firm before it does for the weaker firm. Yet, with N players (and $\Pi > 1$), we conjecture that a result corresponding to Proposition 10 would require high levels of asymmetry across *all* firms.

In our model, research abilities, and hence the degree of asymmetry across players, are exogenously given. It would be interesting to investigate a setting in which players' abilities grew over time as a function of past research efforts (learning by doing).[14] Furthermore, whether to take out a patent, and thus to make one's findings public, is often a strategic decision, conceivably impacting firms' choices of research avenues. We commend these questions to future research.

References

Aghion P, Bloom N, Blundell R, Griffith R, Howitt P (2005). Competition and Innovation: An Inverted U-Relationship, *The Quarterly Journal of Economics*, 120(2), 701-728.

Akcigit U, Liu Q (2016). The Role of Information in Competitive Experimentation, *Journal of the European Economic Association*, 14(4), 828-870.

Besanko D, Wu J (2013). The Impact of Market Structure and Learning on the Tradeoff Between R&D Competition and Cooperation, *The Journal of Industrial Economics*, LXI, 166-201.

Bessen J, Hunt R (2004). An Empirical Look At Software Patents, *Working paper, Federal Reserve Bank of Philadelphia*.

Bessen J, Maskin E (2009). Sequential innovation, patents, and imitation, *The RAND Journal of Economics*, 40(4), 611-635.

Bhattacharya S, Mookerjee D (1986). Portfolio Choice in Research and Development, *Rand Journal of Economics*, 17, 594-605.

Bimpikis K, Ehsani S, Mostagir M (2019). Designing Dynamic Contests, *Operations Research*, 67(2), 339-356.

Bolton P, Harris C (1999). Strategic Experimentation, *Econometrica*, 67, 349-374.

[13]This insight also applies in a setting without payoff externalities, see Das et al. (2020).

[14]Boyarchenko (2019) analyses a game of strategic experimentation with irreversible action choice in which the hazard rate of a breakthrough is increasing in the time spent pursuing it.

Boulet P, Perriens J, Renaud-Thiery F, Velasquez G (2000). Pharmaceuticals and the WTO TRIPS Agreement: Questions and Answers, *UNAIDS/WHO document*.

Boyarchenko S (2019). Super- and submodularity of stopping games with random observations, *Economic Theory*, 1–40.

Bryan K, Lemus J (2017). The direction of innovation, *Journal of Economic Theory*, 172, 247-272.

Chatterjee K, Evans R (2004). Rivals' Search for Buried Treasure: Competition and Duplication in R&D, *Rand Journal of Economics*, 35, 160-183.

Chen C.H, Ishida J (2018). Hierarchical Experimentation, *Journal of Economic Theory*, 177, 365-404.

Chen C.H, Ishida J, Suen W (2020). Reputation Concerns in Risky Experimentation, forthcoming in the *Journal of the European Economic Association*.

Chen Y, Pan S, Zhang T (2014). (When) Do stronger patents increase continual innovation?. *Journal of Economic Behavior & Organization*, 98, 115-124.

Chen Y, Pan S, Zhang T (2018). Patentability, R&D direction, and cumulative innovation. *International Economic Review*, 59(4), 1969-1993.

Choi J.P, Gerlach H (2014). Selection Biases in Complementary R&D Projects, *Journal of Economics and Management Strategy*, 23(4), 899-924.

Das K, Klein N, Schmid K (2019). Strategic Experimentation with Asymmetric Players, *Economic Theory*, Open Access, available at https://link.springer.com/article/10.1007/s00199-019-01193-9

Dasgupta P, Maskin E (1987). The Simple Economics of Research Portfolios, *The Economic Journal*, 97(387), 581-595.

Dong H, Yuede C.M, Coughlan C.A, Murphy K.M, Csernansky J.G (2009). Effects of Donepezil on Amyloid-β and Synapse Density in the Tg2576 Mouse Model of Alzheimer's Disease, *Brain Research*,1303,169-178.

Ederer F (2013). Incentives for Parallel Innovation, *mimeo, Yale University*.

Galasso A, Schankerman M (2015). Patents and Cumulative Innovation: Causal Evidence from the Courts. *The Quarterly Journal of Economics*, 130(1), 317-369.

Gangopadhyay K, Mondal D (2012). Does Stronger Protection of Intellectual Property Stimulate Innovation? *Economics Letters*, 116(1), 80-82.

Halac M, Kartik N, Liu Q (2016). Optimal Contracts for Experimentation, *The Review of Economic Studies*, 83(3), 1040-1091.

Keller G, Rady S, Cripps M (2005). Strategic Experimentation with Exponential Bandits, *Econometrica*, 73, 39-68.

Klein N (2016). The Importance of Being Honest, *Theoretical Economics*, 11, 773-811.

Klette T, de Meza D (1986). Is the Market Biased Against Risky R&D?, *The RAND Journal of Economics*, 17(1), 133-139.

Lee T, Wilde L (1980). Market Structure and Innovation: A Reformulation, *Quarterly Journal of Economics*, 94, 429-436.

Letina I (2016). The Road not Taken: Competition and the R&D Portfolio, *Rand Journal of Economics*, 47(2), 433-460.

Manso G (2011). Motivating Innovation, *The Journal of Finance*, LXVI 5, 1823-1860.

Moss D (2018). Cholinesterase Inhibitors-The Future of Alzheimer's Therapy?, *SciTech Europa Quarterly*, 27, 208-209.

Negoescu D, Bimpikis K, Brandeau M, Iancu D (2018). Dynamic Learning of Patient Response Types: An Application to Treating Chronic Diseases, *Management Science*, 64(8), 3469-3488.

O'Donoghue T, Scotchmer S, Thisse, J.F (1998). Patent Breadth, Patent Life, and the Pace of Technological Progress. *Journal of Economics & Management Strategy*, 7(1), 1-32.

Pammolli F, Magazzini L, Riccaboni M (2011). The Productivity Crisis in Pharmaceutical R&D, *Nature Reviews Drug Discovery*, 10(6), 428-438.

Qian Y (2007). Do National Patent Laws Stimulate Domestic Innovation In a Global Patenting Environment? A Cross-Country Analysis of Pharmaceutical Patent Protection, 1978-2002, *The Review of Economics and Statistics*, 89(3), 436-453.

Reinganum J (1982). A Dynamic Game of R&D Patent Protection and Competitive Behavior, *Econometrica*, 50, 671-688.

Sakakibara M, & Branstetter L (2001). Do Stronger Patents Induce More Innovation? Evidence from the 1988 Japanese Patent Law Reforms, *The RAND Journal of Economics*, 32(1), 77-100.

Sampat B, Williams, H.L (2019). How Do Patents Affect Follow-on Innovation? Evidence from the Human Genome. *American Economic Review*, 109(1), 203-36.

Schneider J, Wolf C (2019). Switching Forth and Back: Problem Solving under Time Pressure, *available at SSRN*.

Wong T (2018). Free Riding and Duplication in R&D, *Working paper, Aalto University*.

APPENDIX

A Ordinary Differential Equations

We define the following decreasing and convex functions:

$$\mu_i(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r+\lambda_0}{\lambda_i}};$$

$$\mu(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{\lambda_1+\lambda_2}}.$$

Throughout this section, we write C for a constant of integration, which is determined from the specific boundary condition. We furthermore write i and j for the two firms, i.e, $\{i,j\} = \{1,2\}$.

A.1 ODEs in the game with symmetric firms

Cartel's problem:

If $k = 0$ is chosen at belief p, the cartel's payoff satisfies $v(p) = \frac{2\lambda_0}{r+2\lambda_0}$. If the cartel chooses $k = 2$ on an open set of beliefs, its payoff function satisfies the ODE

$$2\lambda_1 p(1-p)v' + (r+2\lambda_1 p)v = 2\lambda_1 p. \tag{12}$$

This is solved by

$$v(p) = \frac{2\lambda_1 p}{r+2\lambda_1} + C\mu(p). \tag{13}$$

The non-cooperative game:

If both firms adopt S, either firm's value function satisfies

$$v(p) = \frac{\lambda_0}{r+2\lambda_0}.$$

If both firms adopt R on an open set of beliefs, either firm's value function satisfies

$$2\lambda_1 p(1-p)v' + (r + 2\lambda_1 p)v = \lambda_1 p \tag{14}$$

on this open set. This ODE is solved by

$$v(p) = \frac{\lambda_1 p}{r + 2\lambda_1} + C\mu(p). \tag{15}$$

Now, suppose firm i adopts method R and firm j adopts method S on an open set of beliefs. Then, firm i's value function satisfies

$$\lambda_1 p(1-p)v_i' + (r + \lambda_0 + \lambda_1 p)v_i = \lambda_0(1-\alpha) + \lambda_1 p\alpha \tag{16}$$

on this open set. This is solved by

$$v_i(p) = \frac{\lambda_0(1-\alpha)}{r+\lambda_0} + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\left[\alpha - \frac{\lambda_0(1-\alpha)}{r+\lambda_0}\right] + C\mu_1(p). \tag{17}$$

By the same token, j's value function satisfies

$$\lambda_1 p(1-p)v_j' + (r + \lambda_0 + \lambda_1 p)v_j = \alpha\lambda_0 + \lambda_1 p(1-\alpha). \tag{18}$$

This is solved by

$$v_j(p) = \frac{\lambda_0\alpha}{r+\lambda_0} + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\left[1 - \alpha - \frac{\lambda_0\alpha}{r+\lambda_0}\right] + C\mu_1(p). \tag{19}$$

A.2 ODEs in the game with asymmetric firms

Cartel's problem:

If $k_1 = k_2 = 0$ at belief p, the cartel's payoff is $v(p) = \frac{2\lambda_0}{r+2\lambda_0}$.

If the cartel chooses $k_1 = 1$ and $k_2 = 0$ on an open set of beliefs, its payoff function satisfies the ODE

$$\lambda_1 p(1-p)v' + (r + \lambda_0 + \lambda_1 p)v = \lambda_0 + \lambda_1 p. \tag{20}$$

This is solved by

$$v(p) = \frac{\lambda_0}{r+\lambda_0} + \frac{r\lambda_1}{(r+\lambda_0)(r+\lambda_0+\lambda_1)}p + C\mu_1(p). \tag{21}$$

If the cartel chooses $k_1 = k_2 = 1$ on an open set of beliefs, its payoff function satisfies the ODE

$$(\lambda_1+\lambda_2)p(1-p)v' + (r + (\lambda_1+\lambda_2)p)v = (\lambda_1+\lambda_2)p. \tag{22}$$

This is solved by

$$v(p) = \frac{\lambda_1 + \lambda_2}{r + \lambda_1 + \lambda_2} p + C\mu(p). \tag{23}$$

Non-cooperative game

Suppose both firms adopt method S. Inserting $k_1 = k_2 = 0$ in (5), we can see that both players' payoff is given by the constant

$$\frac{\lambda_0}{r + 2\lambda_0}. \tag{24}$$

Suppose firm i adopts method R and j adopts method S. Inserting $k_i = 1$ and $k_j = 0$ in (5), we can infer that the payoff function of firm i satisfies the ODE

$$\lambda_i p(1-p)v_i' + (r + \lambda_0 + \lambda_i p)v_i = \lambda_0(1-\alpha) + \lambda_i p\alpha. \tag{25}$$

The solution to the above differential equation is

$$v_i^{rs}(p) = \frac{\lambda_0(1-\alpha)}{r+\lambda_0} + \frac{\lambda_i p}{r + \lambda_0 + \lambda_i}\left[\alpha - \frac{\lambda_0(1-\alpha)}{r+\lambda_0}\right] + C\mu_i(p). \tag{26}$$

Firm j's payoff satisfies

$$\lambda_i p(1-p)v_j' + (r + \lambda_0 + \lambda_i p)v_j = \lambda_0\alpha + \lambda_i p(1-\alpha). \tag{27}$$

The solution to the above differential equation is

$$v_j^{rs}(p) = \frac{\lambda_0\alpha}{r+\lambda_0} + \frac{\lambda_i p}{r + \lambda_0 + \lambda_i}\left[1 - \alpha - \frac{\lambda_0\alpha}{r+\lambda_0}\right] + C\mu_i(p). \tag{28}$$

Finally, consider the situation where both firms adopt method R. Inserting $k_1 = k_2 = 1$ in (5), we can infer that the payoff function of either firm i satisfies the ODE

$$(\lambda_1 + \lambda_2)p(1-p)v_i' + (r + (\lambda_1 + \lambda_2)p)v_i = \big(\lambda_i\alpha + \lambda_j(1-\alpha)\big)\, p. \tag{29}$$

The solution to the above differential equation is

$$v_i^{rr}(p) = \frac{\lambda_i\alpha + \lambda_j(1-\alpha)}{r + \lambda_1 + \lambda_2} p + C\mu(p). \tag{30}$$

B Proof of Proposition 1

The payoff function associated with the policy k^* is v. Since $\frac{\lambda_0}{r+2\lambda_0} - \frac{\lambda_1}{r+2\lambda_1} p_1^* > 0$, we know that for $p \in (p_1^*, 1)$, v is strictly convex. Since v satisfies the value matching condition at $p = p_1^*$, direct computation shows that $v^{'}(p_1^*) = 0$. Hence, v is of class C^1 and strictly increasing for $p \in (p_1^*, 1)$. From the ODE (12), we know that $\lambda_1 p[\frac{1}{2} - v - v^{'}(1-p)] = \frac{r}{2}v$. At $p = p_1^*$, $v = \frac{\lambda_0}{r+2\lambda_0}$. This implies $rv = \lambda_0(1-2v)$. Since v is strictly increasing for $p > p_1^*$, for all $p \in (p_1^*, 1)$, we have $rv > \lambda_0(1-2v) \Rightarrow \lambda_1 p[1-2v-2v^{'}(1-p)] > \lambda_0(1-2v)$. Thus, choosing $k = 2$ solves the Bellman equation. On the other hand, since $v^{'} = 0$ for $p \leq p_1^*$, we have $\lambda_1 p[1-2v-2v^{'}(1-p)] \leq \lambda_0(1-2v)$ for $p \in (0, p_1^*]$. Hence, choosing $k = 0$ satisfies the Bellman equation. This shows that the payoff function associated with the proposed policy satisfies the Bellman equation, and hence constitutes the cartel's value function.

C Proof of Proposition 2

We will show that given firm j ($j = 1, 2$) adopts the method R for $p > p_1^*$ and S for $p \leq p_1^*$, this strategy also constitutes the best response of firm i. Consider $p \leq p_1^*$. In this range, we have $v_i = \frac{\lambda_0}{r+2\lambda_0}$. Given firm j's strategy, i has no incentive to deviate as $\lambda_1 p[1 - \frac{\lambda_0}{r+2\lambda_0}] < \lambda_0[1 - \frac{\lambda_0}{r+2\lambda_0}]$ for $p < p_1^*$. Next, consider the range of beliefs $(p_1^*, 1)$. From the closed-form solution of v_i (see equation (13) in Appendix A.1), we can see that v_i is strictly increasing and convex as $[\frac{\lambda_0}{r+2\lambda_0} - \frac{\lambda_1}{r+2\lambda_1} p_1^*] > 0$. At $p = p_1^*$, $v_i = \frac{2\alpha\lambda_0 + \lambda_1 p(1-2\alpha)}{r+2\lambda_0}$. Since $\alpha \geq \frac{1}{2}$, $\frac{2\alpha\lambda_0 + \lambda_1 p(1-2\alpha)}{r+2\lambda_0}$ is non-increasing in p. This implies that, for all $p > p_1^*$, we have $v_i > \frac{2\alpha\lambda_0 + \lambda_1 p(1-2\alpha)}{r+2\lambda_0}$.

To show uniqueness, consider again the range $p \leq p_1^*$ and suppose that a firm adopts method R for a range of beliefs (p_l, p_h) such that $p_l < p_h \leq p_1^*$. Let $\hat{p} < p_1^*$ be the infimum of such beliefs p_l. Then, $v_j(\hat{p}) = v_i(\hat{p}) = \frac{\lambda_0}{r+2\lambda_0}$. Assume without loss of generality that firm i adopts method R in some right-neighbourhood of $\hat{p}$. By the ODEs (14) and (16), it follows immediately from $\hat{p} < p_1^*$ that $v_i < \frac{\lambda_0}{r+2\lambda_0} \leq \frac{2\alpha\lambda_0 + \lambda_1 p(1-2\alpha)}{r+2\lambda_0}$ to the immediate right of $\hat{p}$, implying i has a profitable deviation in a right-neighbourhood of $\hat{p}$.

Now, consider the range $(p_1^*, 1]$. We shall first show that there cannot be a $\check{p} \in (p_1^*, 1]$ such that $(k_i, k_j)(\check{p}) = (0, 0)$ in any equilibrium. Indeed, suppose to the contrary that this was the case. Then, $v_i(\check{p}) = v_j(\check{p}) = \frac{\lambda_0}{r+2\lambda_0}$. By left-continuity of strategies, there exists some left-neighbourhood $\mathcal{N}$ of $\check{p}$ such that $v_i = v_j = \frac{\lambda_0}{r+2\lambda_0}$ and $v_i' = v_j' = 0$ in this neighbourhood. The Bellman equation (2) now implies that either player has a profitable deviation on $\mathcal{N} \cap (p_1^*, \check{p})$. Now, suppose there is an equilibrium in which it is not the case that $(k_i, k_j) = (1, 1)$ prevails everywhere on $(p_1^*, 1]$. Then, there exists some $\tilde{p} \in (p_1^*, 1]$ and a firm j such that $v_j(\tilde{p}) = \frac{\lambda_0}{r+2\lambda_0}$ and $v_j'(\tilde{p}-) \leq 0$. (14) and (16) imply that we must have $(k_i, k_j)(\tilde{p}) = (1, 0)$. The Bellman equation (2) immediately implies that j has a profitable deviation to the immediate left of $\tilde{p}$.

D Proof of Proposition 3

The policy $k^* = (k_1^*, k_2^*)$ implies the payoff function v (given by (4)). As $C_{rs} > 0$, $v_{rs}(p_1^*) = \frac{2\lambda_0}{r+2\lambda_0}$ and $v'_{rs}(p_1^*) = 0$, $v|_{(0,p_2^*)}$ is C^1, (strictly) increasing and (strictly) convex (on (p_1^*, p_2^*)). By ODEs (20) and (22), we have that $v'_{rs}(p_2^*) = v'_{rr}(p_2^*)$. We shall now show that this smooth pasting at p_2^* implies that $C_{rr} > 0$. Indeed, assume to the contrary that $C_{rr} \leq 0$. As $\mu' < 0$ and $p_2^* < 1$, this implies $v'_{rr}(p_2^*) > \frac{\lambda_1+\lambda_2}{r+\lambda_1+\lambda_2}$. Yet, as $C_{rs} > 0$ and $\mu_1' < 0$, we have that $v'_{rs}(p_2^*) < \frac{r\lambda_1}{(r+\lambda_0)(r+\lambda_0+\lambda_1)} < \frac{\lambda_1+\lambda_2}{r+\lambda_1+\lambda_2}$, a contradiction. Thus, $C_{rr} > 0$, and the payoff function v is C^1, (strictly) increasing and (strictly) convex (on $(p_1^*, 1)$).

On $(0, p_1^*)$, $v = \frac{2\lambda_0}{r+2\lambda_0}$ and $v' = 0$, so that $\lambda_i p(1-v) - \lambda_0(1-v) < 0$, as $p < p_1^* = \frac{\lambda_0}{\lambda_1} < \frac{\lambda_0}{\lambda_2}$. Thus, $k_1^* = k_2^* = 0$ solves the Bellman equation (3) in this range.

For $p \in (p_1^*, p_2^*)$, (20) implies

$$\lambda_1 p[1 - v - v'(1-p)] = (r + \lambda_0)v - \lambda_0.$$

Since $v(p_1^*) = \frac{2\lambda_0}{r+2\lambda_0}$ and v is strictly increasing on (p_1^*, p_2^*), we have $\lambda_1 p[1 - v - v'(1-p)] = (r + \lambda_0)v - \lambda_0 > \lambda_0(1-v)$ for this range of beliefs. Thus, $k_1^* = 1$ solves (3) for these beliefs. By the same token, (20) gives us

$$\lambda_2 p[1 - v - v'(1-p)] = \frac{\lambda_2}{\lambda_1}[(r + \lambda_0)v - \lambda_0].$$

Since v is strictly increasing on (p_1^*, p_2^*) and $v(p_2^*) = v_{rs}(p_2^*) = v_{rr}(p_2^*) = \frac{\lambda_0(\lambda_1+\lambda_2)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$, we have that $v < \frac{\lambda_0(\lambda_1+\lambda_2)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ in this range, and hence

$$\lambda_2 p[1 - v - v'(1-p)] = \frac{\lambda_2}{\lambda_1}[(r + \lambda_0)v - \lambda_0] < \lambda_0(1-v).$$

Hence, $k_2^* = 0$ solves (3) on (p_1^*, p_2^*).

Now, let $p > p_2^*$. As v is strictly increasing, $v(p) > \frac{\lambda_0(\lambda_1+\lambda_2)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} = v(p_2^*) > \frac{\lambda_0(\lambda_1+\lambda_2)}{r\lambda_1+\lambda_0(\lambda_1+\lambda_2)}$. By (22), we have

$$\lambda_2 p[1 - v - v'(1-p)] = \frac{\lambda_2}{\lambda_1 + \lambda_2} rv,$$

and hence $\lambda_i p[1 - v - v'(1-p)] > \lambda_0(1-v)$ $(i = 1, 2)$. Thus, $k_1^* = k_2^* = 1$ solves (3) for $p > p_2^*$.

In conclusion, the payoff function v is C^1, and solves the Bellman equation (3); it is thus the value function, and $k^* = (k_1^*, k_2^*)$ is the optimal policy.

It remains to show that $p_2^* > \frac{\lambda_0}{\lambda_2}$. From (3), we can infer that

$$\lambda_2 p_2^*[1 - v(p_2^*) - (1 - p_2^*)v'(p_2^*)] = \lambda_0[1 - v(p_2^*)]$$

Since $v'(p_2^*) > 0$ and $v(p_2^*) < 1$, we have

$$\lambda_2 p_2^*[1 - v(p_2^*)] > \lambda_2 p_2^*[1 - v(p_2^*) - (1 - p_2^*)v'(p_2^*)] = \lambda_0[1 - v(p_2^*)]$$

$$\Rightarrow p_2^* > \frac{\lambda_0}{\lambda_2}.$$

E Proof of Theorem 1

The proposed policies imply a well-defined law of motion of the posterior belief, and lead to the payoff functions as stated in the theorem.

The constant of integration C_1^{rs} is determined from $v_1^{rs}\left(\frac{\lambda_0}{\lambda_1}\right) = \frac{\lambda_0}{r+2\lambda_0}$, which immediately implies $C_1^{rs} > 0$, as $\lambda_1 > \lambda_0$, and $\alpha \geq \frac{1}{2} > \frac{\lambda_0}{r+2\lambda_0}$. Direct computation shows $v_1^{rs\prime}(\frac{\lambda_0}{\lambda_1}+) = 0$.

By the same token, the constant of integration C_2^{rs} is determined from $v_2^{rs}\left(\frac{\lambda_0}{\lambda_1}\right) = \frac{\lambda_0}{r+2\lambda_0}$. Direct calculation shows that this implies $v_2^{rs\prime}(\frac{\lambda_0}{\lambda_1}+) = 0$, $C_2^{rs} < 0$ if $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, $C_2^{rs} > 0$ if $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$, and $C_2^{rs} = 0$ if $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$. Using the ODEs (27) and (29), together with value matching and the definition of $\hat{p}_2$,[15] establishes smooth pasting at $\hat{p}_2$. Thus, v_2 is continuously differentiable. On $(\hat{p}_2, 1)$, it is strictly decreasing and concave on $(p_1^*, \hat{p}_2)$ if $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, strictly increasing and convex if $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$ and flat at $\frac{\lambda_0}{r+2\lambda_0}$ if $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$. As we shall show below, it is convex on $(\hat{p}_2, 1)$.

We now show that $\hat{p}_2(\alpha)$ is well-defined, i.e. that there exists a unique $\hat{p}_2(\alpha) \in (p_1^*, 1)$ such that $F(\hat{p}_2(\alpha), \alpha) = 0$, where the differentiable function F is defined by

$$F(p, \alpha) = v_2^{rs}(p) - \frac{\lambda_0\alpha(\lambda_1 + \lambda_2) + \lambda_1\lambda_2 p(1 - 2\alpha)}{r\lambda_2 + \lambda_0(\lambda_1 + \lambda_2)}.$$

At $p = p_1^*$, $v_2^{rs}(p) = \frac{\lambda_0}{r+2\lambda_0}$ and $\frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p_1^*(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} = \frac{\alpha\lambda_0\lambda_1+(1-\alpha)\lambda_0\lambda_2}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. Thus, we have

$$\frac{\lambda_0\alpha(\lambda_1 + \lambda_2) + \lambda_1\lambda_2 p(1 - 2\alpha)}{r\lambda_2 + \lambda_0(\lambda_1 + \lambda_2)} - v_2^{rs}(p) = \frac{\lambda_0(\alpha r + (2\alpha - 1)\lambda_0)(\lambda_1 - \lambda_2)}{(r\lambda_2 + \lambda_0(\lambda_1 + \lambda_2))(r + 2\lambda_0)} > 0$$

as $\alpha \geq \frac{1}{2}$. Thus, $F(p_1^*, \alpha) < 0$ for all $\alpha \geq \frac{1}{2}$.

At $p = 1$, we have $v_2^{rs}(p) = \frac{\alpha\lambda_0+(1-\alpha)\lambda_1}{r+\lambda_0+\lambda_1}$ and $\frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} = \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+(1-2\alpha)\lambda_1\lambda_2}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. Let $A = \frac{\alpha\lambda_0+(1-\alpha)\lambda_1}{r+\lambda_0+\lambda_1} - \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+(1-2\alpha)\lambda_1\lambda_2}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. Direct computation shows that A is strictly increasing in α, and at $\alpha = \frac{1}{2}$, $A > 0$. Thus, for all $\alpha \geq \frac{1}{2}$, we have $F(1, \alpha) > 0$.

If $\alpha < \frac{r+\lambda_0}{r+2\lambda_2}$, v_2^{rs} is strictly increasing, while $p \mapsto \frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ is decreasing. Thus, we can conclude that there exists a unique $\hat{p}_2(\alpha) \in (p_1^*, 1)$ such that $F(\hat{p}_2(\alpha), \alpha) = 0$.

[15] We omit the argument of $\hat{p}_2(\alpha)$ whenever it is convenient to do so.

If $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_2}$, both v_2^{rs} and $p \mapsto \frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ are (weakly) decreasing in p. The slope of v_2^{rs} is bounded below by $\frac{\lambda_1}{r+\lambda_0+\lambda_1}(1-\frac{r+2\lambda_0}{r+\lambda_0}\alpha)$, while the slope of $\frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ is $\frac{\lambda_1\lambda_2(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. Since

$$B = \frac{\lambda_1\lambda_2(2\alpha-1)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} - \frac{\lambda_1}{r+\lambda_0+\lambda_1}(\frac{r+2\lambda_0}{r+\lambda_0}\alpha-1)$$

is strictly increasing in α and at $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$ we have $B>0$, we can conclude that, for all $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_0}$, we have $B>0$. Thus, the slope of $p \mapsto \frac{\lambda_0\alpha(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ is strictly lower than the lower bound on the slope of v_2^{rs}, so that we can conclude that there exists a unique $\hat{p}_2(\alpha) \in (p_1^*, 1)$ such that $F(\hat{p}_2(\alpha), \alpha) = 0$.

That $\hat{p}_2(\frac{1}{2}) = p_2^*$ follows immediately from the defining equations. By player 2's Bellman equation (5), smooth pasting at $\hat{p}_2$ implies that $\lambda_2\hat{p}_2(\alpha - v_2(\hat{p}_2) - (1-\hat{p}_2)v_2'(\hat{p}_2)) = \lambda_0(\alpha - v_2(\hat{p}_2))$. As $v_2'(\hat{p}_2) < 0$, $v_2'(\hat{p}_2) = 0$ and $v_2'(\hat{p}_2) > 0$ in the cases $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$ and $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$, respectively, this implies $\hat{p}_2 < \frac{\lambda_0}{\lambda_2}$ if $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, $\hat{p}_2 = \frac{\lambda_0}{\lambda_2}$ if $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$ and $\hat{p}_2 > \frac{\lambda_0}{\lambda_2}$ if $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$.

We shall now show that the cutoff $\hat{p}_2$ is strictly decreasing in α. Direct computation shows that $\frac{\partial F}{\partial p} > 0$, so that the sign of $\frac{d\hat{p}_2}{d\alpha}$ is the opposite of the sign of $\frac{\partial F}{\partial \alpha}$. Direct computation shows that $\frac{\partial F}{\partial \alpha}(p, \alpha)$ is independent of α and a strictly increasing continuous function of p, which is strictly negative at $p = \frac{\lambda_0}{\lambda_1}$ and strictly positive at $p = \frac{\lambda_0}{\lambda_2}$. Thus, there exists a unique $\tilde{p} \in (\frac{\lambda_0}{\lambda_1}, \frac{\lambda_0}{\lambda_2})$ such that $\frac{\partial F}{\partial \alpha}$ switches its sign from negative to positive as p increases to $\tilde{p}$.

Since $\hat{p}_2 \geq \frac{\lambda_0}{\lambda_2}$ for $\alpha \in \left[\frac{1}{2}, \frac{r+\lambda_0}{r+2\lambda_0}\right]$, it follows that $\hat{p}_2$ is strictly decreasing in α in this range. Now suppose there exists an $\hat{\alpha} \in (\frac{r+\lambda_0}{r+2\lambda_0}, 1]$ such that $\hat{p}_2(\hat{\alpha}) = \tilde{p}$. Then, $\frac{\partial F}{\partial \alpha}(\hat{p}_2(\hat{\alpha}), \hat{\alpha}) = 0$. As $\hat{\alpha} > \frac{r+\lambda_0}{r+2\lambda_0}$, $\tilde{p} < \frac{\lambda_0}{\lambda_2}$. Since all higher derivatives of this function of α are also 0 at $\hat{\alpha}$, it follows that $\hat{p}_2(\frac{1}{2}) = \hat{p}_2(\hat{\alpha}) = \tilde{p}$. As, by Proposition 3, $\hat{p}_2(\frac{1}{2}) = p_2^* > \frac{\lambda_0}{\lambda_2}$, we get the following chain of inequalities: $\frac{\lambda_0}{\lambda_2} < \hat{p}_2(\frac{1}{2}) = \hat{p}_2(\hat{\alpha}) = \tilde{p} < \frac{\lambda_0}{\lambda_2}$, a contradiction.

It remains to show that our payoff functions satisfy the Bellman equation (5). First, consider the range $[0, \frac{\lambda_0}{\lambda_1}]$. As $v_i = \frac{\lambda_0}{r+2\lambda_0}$ and $v_i' = 0$ in this range, it is immediate that $k_i = 0$ solves the Bellman equation in this range.

Next, let us consider the range $(\frac{\lambda_0}{\lambda_1}, \hat{p}_2]$. As $v_1 > \frac{\lambda_0}{r+2\lambda_0}$ in this range, $k_1 = 1$ satisfies the Bellman equation. Since $v_2(p) \leq \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ for all $p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2]$ by construction, $k_2 = 0$ satisfies the Bellman equation as well.

Finally, we consider the range of beliefs $(\hat{p}_2, 1]$, and first establish strict convexity of v_2 in this range. To do so, we consider the function $\tilde{v}_2(p) = \frac{\lambda_2\alpha+\lambda_1(1-\alpha)}{r+\lambda_1+\lambda_2}p + \tilde{C}_2\mu(p)$, where the constant $\tilde{C}_2$ is implicitly defined by $\tilde{v}_2(p_1^*) = \frac{\lambda_0}{r+2\lambda_0}$. This immediately implies that $\tilde{C}_2 > 0$. By our previous step, player 2 uniquely best-responds by using the established method in the range $(p_1^*, \hat{p}_2)$, which implies that $v_2 > \tilde{v}_2$ on $(p_1^*, \hat{p}_2)$. Therefore, $v_2(\hat{p}_2) = \frac{\lambda_2\alpha+\lambda_1(1-\alpha)}{r+\lambda_1+\lambda_2}\hat{p}_2 + C_2^{rr}\mu(\hat{p}_2) \geq \tilde{v}_2(\hat{p}_2)$, Thus, $C_2^{rr} > 0$, and v_2 is strictly convex on $(\hat{p}_2, 1)$.

By convexity of v_2, smooth pasting at $\hat{p}_2$ and the fact that the graph of v_2^{rs} intersects the graph of $p \mapsto \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2(1-2\alpha)p}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ from below , $v_2' > \frac{\lambda_1\lambda_2(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ in the range $(\hat{p}_2, 1)$. This implies that $v_2(p) > \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2(1-2\alpha)p}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ for all p in this range, and hence player 2 is playing a best response at these beliefs as well.

To show the best-response property on $(\hat{p}_2, 1)$ for player 1 as well, we consider the function $\tilde{v_1}(p) = \frac{\lambda_1\alpha+\lambda_2(1-\alpha)}{r+\lambda_1+\lambda_2}p + \tilde{C}_1\mu(p)$, where the constant $\tilde{C}_1$ is implicitly defined by $\tilde{v}_1(\hat{p}_2) = \frac{\lambda_0}{r+2\lambda_0}$. From (29), it follows that, at any belief $\tilde{p}$ such that $\tilde{v}_1(\tilde{p}) = \frac{\lambda_0}{r+2\lambda_0}$, we have $\tilde{v_1}'(\tilde{p}) > 0$ if and only if $\tilde{p} > \frac{r\lambda_0}{\lambda_0(2\alpha-1)(\lambda_1-\lambda_2)+r(\lambda_1\alpha+\lambda_2(1-\alpha))}$. We will now distinguish the cases (1.) $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_0}$ and (2.) $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$. Direct computation shows that $\frac{r\lambda_0}{\lambda_0(2\alpha-1)(\lambda_1-\lambda_2)+r(\lambda_1\alpha+\lambda_2(1-\alpha))} \leq \frac{\lambda_0}{\lambda_1}$ if and only if $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_0}$. As $\hat{p}_2 > \frac{\lambda_0}{\lambda_1}$, we can conclude that, in case (1.), $\tilde{v_1} > \frac{\lambda_0}{r+2\lambda_0}$ for all $p > \hat{p}_2$. Since $v_1^{rr}(\hat{p}_2) > \tilde{v}_1(\hat{p}_2)$ and $v_1^{rr}(1) = \tilde{v}_1(1)$, we can conclude that $v_1^{rr}(p) > \tilde{v}_1(p)$, and hence that player 1 is playing a best response as well, for all $p \in (\hat{p}_2, 1)$. Now, let us turn to case (2.). Direct computation shows that $\frac{\lambda_0}{\lambda_2} > \frac{r\lambda_0}{\lambda_0(2\alpha-1)(\lambda_1-\lambda_2)+r(\lambda_1\alpha+\lambda_2(1-\alpha))}$. Since $\hat{p}_2 > \frac{\lambda_0}{\lambda_2}$ in case (2.), we can infer that $\tilde{v}_1(p) > \frac{\lambda_0}{r+2\lambda_0}$ for all $p > \hat{p}_2$. Since $v_1^{rr}(\hat{p}_2) > \tilde{v}_1(\hat{p}_2)$ and $v_1^{rr}(1) = \tilde{v}_1(1)$, we can again conclude that $v_1^{rr}(p) > \tilde{v}_1(p)$ for $p \in (\hat{p}_2, 1)$. The fact that, for $p > p_1^*$, $\frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_1+\lambda_0(\lambda_1+\lambda_2)} < \frac{\lambda_0}{r+2\lambda_0}$ implies that firm 1 is playing a best response on $(\hat{p}_2, 1)$ in case (2.) as well.

Let $((k_1(p), k_2(p))_{p\in[0,1]}$ be an equilibrium of the game and define $p_l = \inf\{p \in [0,1] : \exists i \in \{1,2\}, k_i = 1\}$. If $p_l > \frac{\lambda_0}{\lambda_1}$, firm 1 has profitable deviation on $(\frac{\lambda_0}{\lambda_1}, p_l)$. Thus, $p_l \leq \frac{\lambda_0}{\lambda_1}$.

Suppose that $p_l < \frac{\lambda_0}{\lambda_1}$. There are now two possibilities. (*i*) First, suppose both firms are using R to the immediate right of p_l. Note that, for any $p < \frac{\lambda_0}{\lambda_1}$, we have $\frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} > \frac{\lambda_0[\alpha\lambda_1+(1-\alpha)\lambda_2]}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} > \frac{\lambda_0}{r+2\lambda_0} > 0$. As payoffs are continuous and both firms' payoff at $p = p_l$ is equal to $\frac{\lambda_0}{r+2\lambda_0}$, we will have

$$\frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)} > v_2(p)$$

in some right-neighbourhood of p_l, and thus firm 2 is not playing a best response-a contradiction. Thus, suppose that (*ii*) only one of the firms, firm i, is using R at beliefs just above p_l. As $p_l < \frac{\lambda_0}{\lambda_1} < \frac{\lambda_0}{\lambda_2}$, (25) implies that $v_i' < 0$ for beliefs just above p_l. This implies that v_i drops below $\frac{\lambda_0}{r+2\lambda_0}$ in some right-neighbourhood of p_l, implying that firm i is not playing a best response there. We thus conclude that $p_l = \frac{\lambda_0}{\lambda_1}$.

We will now establish that, in any equilibrium, there exists a right-neighbourhood of $\frac{\lambda_0}{\lambda_1}$ in which firm 1 plays R while firm 2 plays S. First, suppose to the contrary that both firms play R just above $\frac{\lambda_0}{\lambda_1}$. Then, by the same argument as above, $v_2 < \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$ for some beliefs just above $\frac{\lambda_0}{\lambda_1}$, implying that firm 2 is not playing a best response there. By the same token, it is not possible that only firm 2 uses R in equilibrium to the immediate right of $\frac{\lambda_0}{\lambda_1}$, because, by (25), the payoff of firm 2 would fall below $\frac{\lambda_0}{r+2\lambda_0}$–a contradiction. We have thus established that, in any equilibrium, firm 1 will play R while firm 2 will play S in some right-neighbourhood of $\frac{\lambda_0}{\lambda_1}$.

For the range $(p_1^*, \hat{p}_2]$, we shall distinguish two cases: (1.) $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_0}$ and (2.) $\alpha < \frac{r+\lambda_0}{r+2\lambda_0}$. We start with case (1.), and shall argue next that, in no equilibrium, there exists a $p^{'} \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2)$ such that to the immediate right of $p^{'}$, firm 2 uses the method R and firm 1 uses S. Suppose to the contrary that such a p' exists and let $p_l^{'}$ be the lowest of such beliefs p'. Then, if $\alpha > \frac{r+\lambda_0}{r+2\lambda_0}$, the payoff function of firm 2 (26) is strictly less than $\frac{\lambda_0}{r+2\lambda_0}$ to the immediate right of $p_l^{'}$, implying that firm 2 is not playing a best response. If $\alpha = \frac{r+\lambda_0}{r+2\lambda_0}$, $p^{'} < \hat{p}_2 = \frac{\lambda_0}{\lambda_2}$ implies that the payoff function of firm 2 (26) drops below $\frac{\lambda_0}{r+2\lambda_0}$ to the immediate right of $p^{'}$, so that firm 2 is not playing a best response in some right-neighbourhood of $p^{'}$.

By the same token, let p_l'' be the lowest belief in $(\frac{\lambda_0}{\lambda_1}, \hat{p}_2)$ such that both firms use method S in some right-neighbourhood of p_l''. We have already established that, in any equilibrium, either $(k_1, k_2) = (1,0)$ or $(k_1, k_2) = (1,1)$ prevails throughout $(p_1^*, p_l'']$. Using the ODEs (25) and (29) and the assumption $\alpha \geq \frac{r+\lambda_0}{r+2\lambda_0}$, one can show that firm 1's payoff satisfies $v_1(p_l''-) > \frac{\lambda_0}{r+2\lambda_0}$, implying firm 1 has a profitable deviation.

Now, let p_l''' be the lowest belief in $(\frac{\lambda_0}{\lambda_1}, \hat{p}_2)$ such that both firms use method R in some right-neighbourhood of p_l'''. Then, firm 2's payoff satisfies $v_2(p_l''') = v_2^{rs}(p_l''') < \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$, where the inequality follows from $p_l''' < \hat{p}_2$, implying that firm 2 has a profitable deviation.

Now, let us turn to case (2.) and suppose there exists an equilibrium with the feature that there exists a $p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2)$ such that $(k_1, k_2) \neq (1,0)$, and let $p_l = \inf\{p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2) : (k_1, k_2) \neq (1,0)\}$. Thus, $v_1(p_l) = v_1^{rs}(p_l) > \frac{\lambda_0}{r+2\lambda_0} > \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_1+\lambda_0(\lambda_1+\lambda_2)}$ and $v_2(p_l) = v_2^{rs}(p_l) < \frac{\alpha\lambda_0(\lambda_1+\lambda_2)+\lambda_1\lambda_2 p(1-2\alpha)}{r\lambda_2+\lambda_0(\lambda_1+\lambda_2)}$. This implies that $k_1 = 1$ is a strictly dominant action for firm 1 in some right-neighbourhood of p_l, while $k_2 = 0$ is firm 2's unique best response in this range, a contradiction. We have thus established that, in any equilibrium, for $p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2)$, firm 1 uses R and 2 uses S.

We shall now argue that for all $p > \hat{p}_2$, using method R is the dominant action for firm 1. Suppose not and let $\tilde{p}$ be the lowest belief in $(\hat{p}_2, 1)$ such that firm 1 uses S while firm 2 uses R in some right-neighbourhood of $\tilde{p}$. Our verification arguments imply that firm 1 is not playing a best response at beliefs just above $\tilde{p}$. A similar argument to above furthermore establishes that firm 1 would have a profitable deviation at the lowest belief $\tilde{p}' \in (\hat{p}_2, 1)$ such that both firms use S is some right-neighbourhood of $\tilde{p}'$. This shows that for all $p > \hat{p}_2$, using method R is the dominant action of firm 1. From our equilibrium construction, it follows that the unique best response of firm 2 is to choose R, which concludes the proof.

F Payoff-Functions and Best-reponses for the General model

Closed-Form Expressions

We fix a belief $p \in (0,1)$ and assume player j's action is constant in some neighbourhood of p. (By left-continuity of strategies, this will be the case at a.a. p.) C denotes a constant of integration.

Let $k_0^{(j)} = k_1^{(j)} = 0$ in a neighbourhood of p

If i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$,

$$v_i(p) = 0.$$

If i's best response is given by $k_0^{(i)} = 1$,

$$v_i(p) = \frac{\lambda_0}{r+\lambda_0}\frac{A^2(10\alpha-1)}{36B} - \frac{s}{r+\lambda_0}.$$

Remark 1 *This implies immediately that, if $\frac{A^2(10\alpha-1)}{36B} > \frac{s}{\lambda_0}$, a breakthrough will occur almost surely as $t \to \infty$ in any equilibrium.*

[The corresponding condition for the cartel's (the utilitarian planner's) problem is $\frac{s}{\lambda_0} < \frac{A^2}{4B}$ ($\frac{s}{\lambda_0} < \frac{A^2}{2B}$).

Moreover, $\frac{A^2(10\alpha-1)}{36B} < \frac{A^2}{4B}$ ($\frac{A^2(10\alpha-1)}{36B} = \frac{A^2}{4B}$) if and only if $\alpha < 1$ ($\alpha = 1$). Thus, if and only if $\alpha < 1$, there exists a range of parameters where players give up with positive probability, while the cartel would not. There always exists a range of parameters for which the cartel gives up with positive probability, while a utilitarian planner would not.]

If i's best response is given by $k_1^{(i)} = 1$, v_i satisfies the ODE

$$\lambda_i p(1-p)v_i' + (r+p\lambda_i)v_i = p\lambda_i \frac{A^2(10\alpha-1)}{36B}\Pi - s,$$

which is solved by

$$v_i(p) = -\frac{s}{r}\left(1 - p\frac{\lambda_i}{r+\lambda_i}\right) + p\frac{\lambda_i}{r+\lambda_i}\frac{A^2(10\alpha-1)}{36B}\Pi + C\bar{\mu}_i(p),$$

where we write $\bar{\mu}_i(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{\lambda_i}}$.

Let $k_0^{(j)} = 1$ in a neighbourhood of p

If i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$,

$$v_i(p) = \frac{\lambda_0}{r+\lambda_0}\frac{2A^2(1-\alpha)}{9B}.$$

If i's best response is given by $k_0^{(i)} = 1$,

$$v_i(p) = \frac{\lambda_0}{r+2\lambda_0}\frac{A^2(7+2\alpha)}{36B} - \frac{s}{r+2\lambda_0}.$$

If i's best response is given by $k_1^{(i)} = 1$, v_i satisfies the ODE

$$\lambda_i p(1-p)v_i' + (r+\lambda_0+\lambda_i p)v_i = \lambda_i p\frac{A^2(10\alpha-1)}{36B}\Pi + \lambda_0\frac{2A^2(1-\alpha)}{9B} - s,$$

which is solved by

$$v_i(p) = \left(\frac{\lambda_0}{r+\lambda_0}\frac{2A^2(1-\alpha)}{9B} - \frac{s}{r+\lambda_0}\right)\left(1 - \frac{p\lambda_i}{r+\lambda_0+\lambda_i}\right) + \frac{p\lambda_i\Pi}{r+\lambda_0+\lambda_i}\frac{A^2(10\alpha-1)}{36B} + C\mu_i(p),$$

where $\mu_i(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r+\lambda_0}{\lambda_i}}$.

Let $k_1^{(j)} = 1$ in a neighbourhood of p

If i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$, v_i satisfies the ODE

$$\lambda_j p(1-p)v_i' + (r+p\lambda_j)v_i = p\lambda_j\frac{2A^2(1-\alpha)}{9B}\Pi,$$

which is solved by

$$v_i(p) = \frac{\lambda_j}{\lambda_j+r}p\frac{2A^2(1-\alpha)}{9B}\Pi + C\bar{\mu}_j(p).$$

If i's best response is given by $k_0^{(i)} = 1$, v_i satisfies the ODE

$$\lambda_j p(1-p)v_i' + (r+\lambda_0+\lambda_j p)v_i = \lambda_0\frac{A^2(10\alpha-1)}{36B} + \lambda_j p\frac{2A^2(1-\alpha)}{9B}\Pi - s,$$

which is solved by

$$v_i(p) = \frac{1}{r+\lambda_0}\left(\lambda_0 \frac{A^2(10\alpha-1)}{36B} - s\right)\left(1 - \frac{\lambda_j p}{r+\lambda_0+\lambda_j}\right) + \frac{\lambda_j p}{r+\lambda_0+\lambda_j}\frac{2A^2(1-\alpha)}{9B}\Pi + C\mu_j(p).$$

If i's best response is given by $k_1^{(i)} = 1$, v_i satisfies the ODE

$$(\lambda_i+\lambda_j)p(1-p)v_i' + (r+(\lambda_i+\lambda_j)p)v_i = p\Pi\left[\lambda_i \frac{A^2(10\alpha-1)}{36B} + \lambda_j \frac{2A^2(1-\alpha)}{9B}\right] - s,$$

which is solved by

$$v_i(p) = -\frac{s}{r}\left(1 - \frac{p(\lambda_i+\lambda_j)}{r+\lambda_i+\lambda_j}\right) + \frac{p\Pi}{r+\lambda_i+\lambda_j}\left[\lambda_i \frac{A^2(10\alpha-1)}{36B} + \lambda_j \frac{2A^2(1-\alpha)}{9B}\right] + C\mu(p),$$

where $\mu(p) = (1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{\lambda_i+\lambda_j}}$.

Best-Response Analysis

Let $k_0^{(j)} = k_1^{(j)} = 0$ in a neighbourhood of p

i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$ if and only if

$$\lambda_0 \frac{A^2(10\alpha-1)}{36B} \leq s$$

and

$$p \leq \frac{s}{\lambda_i \frac{A^2\Pi(10\alpha-1)}{36B}}.$$

i's best response is given by $k_0^{(i)} = 1$ if and only if

$$\lambda_0 \frac{A^2(10\alpha-1)}{36B} \geq s$$

and

$$p \leq \frac{\lambda_0}{\lambda_i}\frac{r}{r+\lambda_0}\frac{\frac{A^2(10\alpha-1)}{36B} + \frac{s}{r}}{\left(\Pi - \frac{\lambda_0}{r+\lambda_0}\right)\frac{A^2(10\alpha-1)}{36B} + \frac{s}{r+\lambda_0}}.$$

i's best response is given by $k_1^{(i)} = 1$ if and only if

$$v_i(p) \geq \max\left\{0, \frac{\lambda_0}{r+\lambda_0}\frac{A^2(10\alpha-1)}{36B} - \frac{s}{r+\lambda_0}\right\}.$$

Let $k_0^{(j)} = 1$ in a neighbourhood of p

i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$ if and only if

$$\frac{\lambda_0}{r+\lambda_0}\frac{A^2}{36B}\left((10\alpha-1)r + (18\alpha-9)\lambda_0\right) \leq s$$

and

$$p \leq \frac{s}{\lambda_i \frac{A^2}{9B}\left(\frac{\Pi(10\alpha-1)}{4} - \frac{2\lambda_0(1-\alpha)}{r+\lambda_0}\right)}.$$

i's best response is given by $k_0^{(i)} = 1$ if and only if

$$\frac{\lambda_0}{r+\lambda_0}\frac{A^2}{36B}\left((10\alpha-1)r + (18\alpha-9)\lambda_0\right) \geq s$$

and

$$p \leq \frac{\lambda_0}{\lambda_i}\frac{\frac{A^2(10\alpha-1)}{36B} - \frac{\lambda_0}{r+2\lambda_0}\frac{A^2(7+2\alpha)}{36B} + \frac{s}{r+2\lambda_0}}{\Pi\frac{A^2(10\alpha-1)}{36B} - \frac{\lambda_0}{r+2\lambda_0}\frac{A^2(7+2\alpha)}{36B} + \frac{s}{r+2\lambda_0}} = \frac{\lambda_0}{\lambda_1}\frac{A^2[9\lambda_0(2\alpha-1) + r(10\alpha-1)] + 36Bs}{A^2[\lambda_0(-7-2\alpha-2\Pi+20\alpha\Pi) + \Pi r(10\alpha-1)] + 36Bs}.$$

i's best response is given by $k_1^{(i)} = 1$ if and only if

$$v_i(p) \geq \max\left\{\frac{\lambda_0}{r+\lambda_0}\frac{2A^2(1-\alpha)}{9B}, \frac{\lambda_0}{r+2\lambda_0}\frac{A^2(7+2\alpha)}{36B} - \frac{s}{r+2\lambda_0}\right\}.$$

Let $k_1^{(j)} = 1$ in a neighbourhood of p

i's best response is given by $k_0^{(i)} = k_1^{(i)} = 0$ if and only if

$$\frac{A^2(10\alpha-1)}{36B} - \frac{s}{\lambda_0} \leq v_i(p) \leq \frac{\lambda_j}{\lambda_i}\frac{s}{r} - \frac{\lambda_j}{r}p\frac{A^2\Pi}{4B}(2\alpha-1).$$

i's best response is given by $k_0^{(i)} = 1$ if and only if

$$v_i(p) \leq \min\left\{\frac{A^2(10\alpha-1)}{36B} - \frac{s}{\lambda_0}, \frac{A^2}{36B}\frac{\lambda_0(\lambda_i+\lambda_j)(10\alpha-1) - \lambda_i\lambda_j p\Pi(18\alpha-9)}{r\lambda_i + \lambda_0(\lambda_i+\lambda_j)} - \frac{\lambda_i}{r\lambda_i+\lambda_0(\lambda_i+\lambda_j)}s\right\}.$$

i's best response is given by $k_1^{(i)} = 1$ if and only if

$$v_i(p) \geq \max\left\{ \frac{\lambda_j}{\lambda_i}\frac{s}{r} - \frac{\lambda_j}{r}p\frac{A^2\Pi}{4B}(2\alpha-1), \frac{A^2}{36B}\frac{\lambda_0(\lambda_i+\lambda_j)(10\alpha-1)-\lambda_i\lambda_j p\Pi(18\alpha-9)}{r\lambda_i+\lambda_0(\lambda_i+\lambda_j)} - \frac{\lambda_i}{r\lambda_i+\lambda_0(\lambda_i+\lambda_j)}s \right\}$$

.

G Proof of Proposition 5

We begin the proof by observing that $\frac{s}{\lambda_0} < \frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$ implies $\frac{s}{\lambda_0} < \frac{A^2}{9B}$, so that for all $\alpha \in (\frac{1}{2}, 1]$, we have $\frac{s}{\lambda_0} < \frac{A^2}{36B}(10\alpha-1)$. Hence, for all levels of patent protection, the established method S is not dominated. Further, $\frac{s}{\lambda_0} < \frac{A^2}{9B}\frac{\lambda_2 r}{\lambda_0\lambda_1+\lambda_2 r}$ also ensures that, for a given level of patent protection α, if firm j is using R at a belief p, then it is optimal for firm $i \neq j$ to use R at p as well, if and only if $v_i(p) \geq \psi_i(p)$ where

$$\psi_i(p) = \frac{A^2}{36B}\frac{\lambda_0(\lambda_1+\lambda_2)(10\alpha-1)-9\lambda_1\lambda_2 p(2\alpha-1)}{r\lambda_i+\lambda_0(\lambda_1+\lambda_2)} - \frac{\lambda_i s}{r\lambda_i+\lambda_0(\lambda_1+\lambda_2)}. \tag{31}$$

On the other hand, for a given value of α, if firm j is using S at p, then it is optimal for firm i to use R at p if and only if $v_i(p) \geq v_{ss}$, where

$$v_{ss} = \frac{\lambda_0}{r+2\lambda_0}\frac{A^2}{36B}(7+2\alpha) - \frac{s}{r+2\lambda_0} \tag{32}$$

denotes the payoff to a firm when both firms adopt the method R. The proposed strategies imply a well-defined law of motion of the posterior belief, and lead to the following payoff functions:

$$v_1(p) = \begin{cases} -\frac{s}{r}(1-\frac{p(\lambda_1+\lambda_2)}{r+\lambda_1+\lambda_2}) + \frac{p}{r+\lambda_1+\lambda_2}[\lambda_1\frac{A^2}{36B}(10\alpha-1)+\lambda_2\frac{2A^2(1-\alpha)}{9B}] + C_1^{rr}\mu(p) & \equiv v_1^{rr}(p) \\ & \text{if } p \in (p_2(\alpha), 1] \\ (\frac{\lambda_0}{r+\lambda_0}\frac{2A^2(1-\alpha)}{9B} - \frac{s}{r+\lambda_0})(1-\frac{p\lambda_1}{r+\lambda_0+\lambda_1}) + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\frac{A^2(10\alpha-1)}{36B} + C_1^{rs}\mu_1(p) & \equiv v_1^{rs}(p) \\ & \text{if } p \in (\frac{\lambda_0}{\lambda_1}, p_2(\alpha)] \\ \frac{A^2}{36B}\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha) - \frac{s}{r+2\lambda_0} & \equiv v^{ss} \\ & \text{if } p \in (0, \frac{\lambda_0}{\lambda_1}], \end{cases} \tag{33}$$

$$v_2(p)=\begin{cases} -\frac{s}{r}(1-\frac{p(\lambda_1+\lambda_2)}{r+\lambda_1+\lambda_2})+\frac{p}{r+\lambda_1+\lambda_2}[\lambda_2\frac{A^2}{36B}(10\alpha-1)+\lambda_1\frac{2A^2(1-\alpha)}{9B}]+C_2^{rr}\mu(p) & \equiv v_2^{rr}(p) \\ & \text{if } p\in(p_2(\alpha),1] \\ (\frac{\lambda_0}{r+\lambda_0}\frac{A^2(10\alpha-1)}{36B}-\frac{s}{r+\lambda_0})(1-\frac{p\lambda_1}{r+\lambda_0+\lambda_1})+\frac{\lambda_1 p}{r+\lambda_0+\lambda_1}\frac{2A^2(1-\alpha)}{9B}+C_2^{rs}\mu_1(p) & \equiv v_2^{rs}(p) \\ & \text{if } p\in(\frac{\lambda_0}{\lambda_1},p_2(\alpha)] \\ \frac{A^2}{36B}\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha)-\frac{s}{r+2\lambda_0} & \equiv v^{ss} \\ & \text{if } p\in(0,\frac{\lambda_0}{\lambda_1}], \end{cases} \tag{34}$$

v_1^{rs} satisfies the following ODE:

$$\lambda_1 p(1-p)v_1^{'}+(r+\lambda_0+\lambda_1 p)v_1=\lambda_1 p\frac{A^2}{36B}(10\alpha-1)+\lambda_0\frac{2A^2}{9B}(1-\alpha)-s.$$

The constant of integration C_1^{rs} is determined from $v_1^{rs}(\frac{\lambda_0}{\lambda_1})=v_{ss}$. This immediately implies $C_1^{rs}>0$ as $\lambda_1>\lambda_0$ and $\alpha\geq\frac{1}{2}$. Direct computation shows that $v_1^{rs'}(\frac{\lambda_0}{\lambda_1}+)=0$.

The value matching condition $v_2^{rs}(\frac{\lambda_0}{\lambda_1})=v_{ss}$ determines the constant of integration C_2^{rs}. v_2^{rs} satisfies the ODE:

$$\lambda_1 p(1-p)v_2^{'}+(r+\lambda_0+\lambda_1 p)v_2=\lambda_0[\frac{A^2}{36B}(10\alpha-1)]+\lambda_1 p[\frac{2A^2}{9B}(1-\alpha)]-s.$$

Direct computation shows that $v_2^{rs'}(\frac{\lambda_0}{\lambda_1}+)=0$, as well as $C_2^{rs}<0$ if $\alpha>\bar{\alpha}$, $C_2^{rs}>0$ if $\alpha<\bar{\alpha}$ and $C_2^{rs}=0$ if $\alpha=\bar{\alpha}$ where $\bar{\alpha}=\frac{8r+9\lambda_0}{8r+18\lambda_0}+\frac{36B}{A^2}\frac{s}{8r+18\lambda_0}$.

Let p_2 be the lowest belief $p\in\left(\frac{\lambda_0}{\lambda_1},1\right)$ such that $F(p,\alpha):=v_2^{rs}(p)-\psi_2(p)=0$. We will now show that p_2 is well defined, i.e., that there exists a $p_2\in(\frac{\lambda_0}{\lambda_1},1)$ such that $F(p_2,\alpha)=0$. Indeed, by direct computation, $F(\frac{\lambda_0}{\lambda_1},\alpha)<0<F(1,\alpha)$, for any $\alpha\in[\frac{1}{2},1]$. By continuity of F, the intermediate value theorem implies there exists a $p_2\in(\frac{\lambda_0}{\lambda_1},1)$ such that $F(p_2,\alpha)=0$. As μ_1 is strictly convex, v_2^{rs} is [strictly] concave (convex) if and only if $\alpha\geq\bar{\alpha}$ ($\alpha\leq\bar{\alpha}$) [whenever the inequality is strict]. Further, $v_2^{rs'}(\frac{\lambda_0}{\lambda_1})=0\geq\psi_2^{'}$, with a strict inequality for $\alpha>\frac{1}{2}$, and $v_2^{rs'}(1)>\psi_2^{'}$. It follows that v_2^{rs} is [strictly] decreasing (increasing) on $\left(\frac{\lambda_0}{\lambda_1},1\right)$ if and only if $\alpha\geq\bar{\alpha}$ ($\alpha\leq\bar{\alpha}$) [whenever the inequality is strict]. Thus, the point p_2 is unique if $\alpha>\frac{1}{2}$. Furthermore, the point of intersection p_2 is also unique if $\alpha=\frac{1}{2}$, as v_2^{rs} is strictly convex, and thus strictly increasing on $\left(\frac{\lambda_0}{\lambda_1},1\right)$, in this case.

For $p\in(p_2,1)$, $v_2=v_2^{rr}$ and v_2 satisfies the ODE

$$(\lambda_1+\lambda_2)p(1-p)v_2^{'}+(r+(\lambda_1+\lambda_2)p)v_2=p\left[\lambda_2\frac{A^2}{36B}(10\alpha-1)+\lambda_1 p\frac{2A^2}{9B}(1-\alpha)\right]-s. \tag{35}$$

As we have seen above, v_2^{rs} is increasing (decreasing) if and only if $\alpha \leq \bar{\alpha}$ ($\alpha \geq \bar{\alpha}$). By (35), this observation directly implies that $p_2 \geq \frac{\lambda_0}{\lambda_2}$ ($p_2 \leq \frac{\lambda_0}{\lambda_2}$) if and only if $\alpha \leq \bar{\alpha}$ ($\alpha \geq \bar{\alpha}$), with equality if and only if $\alpha = \bar{\alpha}$.

By the relevant ODEs, direct computation shows that v_2 is smooth at $p = p_2$, i.e $v_2^{rs'}(p_2) = v_2^{rr'}(p_2)$. By the definition of p_2, v_2 intersects ψ_2 from below at p_2. The closed-form expression for v_2^{rr} implies that it is either globally concave or globally convex. Since $v_2^{rr'}(1) > 0 \geq \psi_2'$, v_2 is always above ψ_2 for all $p > p_2$. This establishes that firm 2 is playing a best response on $(p_2, 1]$ in the conjectured equilibrium.

We will now establish the best response of firm 1. For $p \in (0, \frac{\lambda_0}{\lambda_1})$, $v_1 = \frac{\lambda_0}{r+2\lambda_0}\frac{A^2}{36B}(10\alpha - 1) \equiv v^{ss}$, implying firm 1 is playing a best response. Since v_1^{rs} is strictly convex and $v_1^{rs'}(p_1) = 0$, for all $p \in (\frac{\lambda_0}{\lambda_1}, p_2]$, $v_1 = v_1^{rs} > \frac{\lambda_0}{r+2\lambda_0}\frac{A^2}{36B}(10\alpha - 1)$. This implies that firm 1's action constitutes a best response on $p \in (p_1, p_2]$. Finally, we consider the range $(p_2, 1]$. Define $\tilde{v_1}(p)$ by

$$\tilde{v_1}(p) = -\frac{s}{r}\left(1 - \frac{p(\lambda_1 + \lambda_2)}{r + \lambda_1 + \lambda_2}\right)\frac{p}{r + \lambda_1 + \lambda_2}[\lambda_1 \frac{A^2}{36B}(10\alpha - 1) + \lambda_2 \frac{2A^2}{9B}(1 - \alpha)] + \tilde{C}\mu(p),$$

where $\tilde{C}$ is such that $\tilde{v_1}(p_2) = v^{ss}$. If $\tilde{v_1}(p') = v^{ss}$, then, by the relevant ODE, $\tilde{v_1}'(p') \geq 0$ if and only if $p' \geq \check{p}(\alpha)$, where

$$\check{p}(\alpha) := \frac{r\lambda_0(7 + 2\alpha) + \frac{72B}{A^2}\lambda_0 s}{9\lambda_0(2\alpha - 1)(\lambda_1 - \lambda_2) + r(\lambda_1(10\alpha - 1) + \lambda_2 8(1 - \alpha)) + \frac{36B}{A^2}(\lambda_1 + \lambda_2)s}.$$

One verifies that $\check{p}(\alpha) < \frac{\lambda_0}{\lambda_2}$ for all $\alpha \in [\frac{1}{2}, 1]$, and that $\check{p}(\alpha) < \frac{\lambda_0}{\lambda_1}$ if and only if $\alpha > \bar{\alpha}$. As for $\alpha \leq \bar{\alpha}$, $p_2 \geq \frac{\lambda_0}{\lambda_2}$, we can conclude that $\tilde{v}_1'(p_2) > 0$. Since $v_1^{rr}(p_2) = v_1^{rs}(p_2) > v^{ss}$, we can conclude that $v_1^{rr} > \tilde{v_1}$ on $[p_2, 1)$.

Consider first the case $\alpha > \bar{\alpha}$. We know that v_2^{rs} is strictly decreasing on $(p_1, p_2]$. Since $v_2^{rs}(p_2) = \psi_2(p_2)$, we have $v_1^{rs} > v^{ss} = \tilde{v_1} > v_2^{rs} = \psi_2(p_2) > \psi_1(p_2)$ as $\lambda_1 > \lambda_2$. As ψ_1 is strictly decreasing in p, we have $\tilde{v_1} > \psi_1$ for all $p > p_2$. This implies $v_1^{rr} > \tilde{v_1} > \psi_1$ for all $p > p_2$.

Next, consider $\alpha \leq \bar{\alpha}$. Direct computation shows that $v^{ss} - \psi_1(\frac{\lambda_0}{\lambda_1})$ is strictly decreasing in α and $v^{ss} - \psi_1(\frac{\lambda_0}{\lambda_1}) = 0$ at $\alpha = \bar{\alpha}$. This implies that, for all $\alpha \leq \bar{\alpha}$, $v^{ss} - \psi_1(\frac{\lambda_0}{\lambda_1}) \geq 0$.

Since ψ_1 is strictly decreasing in p, $\tilde{v_1}(p_2) = v^{ss} > \psi_1(p_2)$, and, therefore, $v_1^{rr}(p) \geq \tilde{v_1}(p) > \psi_1(p)$ for all $p \in [p_2, 1]$.

Direct computation shows that $\frac{\partial F}{\partial p} > 0$, so that the sign of $\frac{dp_2}{d\alpha}$ is the opposite of the sign of $\frac{\partial F}{\partial \alpha}$. Direct computation shows that $\frac{\partial F}{\partial \alpha}(p, \alpha)$ is independent of α and a strictly increasing continuous function of p, which is strictly negative at $p = \frac{\lambda_0}{\lambda_1}$, and strictly positive at $p = \frac{\lambda_0}{\lambda_2}$. Thus, there exists a unique $\tilde{p} \in \left(\frac{\lambda_0}{\lambda_1}, \frac{\lambda_0}{\lambda_2}\right)$ such that $\frac{\partial F}{\partial \alpha}$ switches its sign from negative to positive as p increases to $\tilde{p}$. Now suppose there exists an $\hat{\alpha} \in (\bar{\alpha}, 1]$ such that $p_2(\hat{\alpha}) = \tilde{p}$. Then, $\frac{\partial F}{\partial \alpha}(p_2(\hat{\alpha}), \hat{\alpha}) = 0$. As $\hat{\alpha} > \bar{\alpha}$, $\tilde{p} < \frac{\lambda_0}{\lambda_2}$.

Since all higher derivatives of this function of α are also 0 at $\hat{\alpha}$, it follows that $p_2(\frac{1}{2}) = p_2(\hat{\alpha}) = \tilde{p}$. Yet, as $\bar{\alpha} > \frac{1}{2}$, $p_2(\frac{1}{2}) > \frac{\lambda_0}{\lambda_2}$, this leads to the following chain of inequalities: $\frac{\lambda_0}{\lambda_2} < p_2(\frac{1}{2}) = p_2(\hat{\alpha}) = \tilde{p} < \frac{\lambda_0}{\lambda_2}$, a contradiction.

H Proof of Proposition 6

Proof is constructive. We are constructing an equilibrium in cutoff strategies with the same structure as in our baseline model (Theorem 1). In this putative equilibrium, the more productive firm uses a more pessimistic threshold. Thus, there exist thresholds $\underline{p} < \bar{p}$ such that both firms use method S on $[0, \underline{p}]$, firm 1 (2) uses R (S) on $p \in (\underline{p}, \bar{p}]$ and both firms use R on $(\bar{p}, 1]$. We conjecture $\underline{p} = \frac{s}{\lambda_1 \frac{A^2(10\alpha-1)}{36B}}$. It remains to show that these actions constitute mutual best responses. By substituting $\Pi = 1$ in (9) we obtain the best response line for player i (against a player j who uses method R) as $\psi_i = \frac{\lambda_j}{\lambda_i}\frac{s}{r} - \frac{\lambda_j}{r} p \frac{A^2}{4B}(2\alpha - 1)$.

In the conjectured equilibrium, for $p \in (\underline{p}, \bar{p}]$, the payoff of firm 1 is given by

$$v_1^{rs}(p) = -\frac{s}{r}\left(1 - p\frac{\lambda_1}{r+\lambda_1}\right) + p\frac{\lambda_1}{r+\lambda_1}\frac{A^2(10\alpha-1)}{36B} + C\bar{\mu}_1(p),$$

where C is a constant of integration defined by $v_1^{rs}(\underline{p}) = 0$. Direct computation shows that the smooth-pasting condition $v_1^{rs'}(\underline{p}) = 0$ is satisfied for $\underline{p} = \frac{s}{\lambda_1 \frac{A^2(10\alpha-1)}{36B}}$, and that v_1^{rs} is strictly convex. Firm 2's payoff in this range is given by

$$v_2^{rs}(p) = \frac{\lambda_1}{\lambda_1 + r}\frac{2A^2(1-\alpha)}{9B}\left[p - \underline{p}\frac{\bar{\mu}_1(p)}{\bar{\mu}_1(\underline{p})}\right].$$

v_2^{rs} is (strictly) increasing (if $\alpha < 1$), while ψ_2 is strictly decreasing, in p. Since at $\bar{p}$, firm 2 optimally decides to start researching using R, we have $\psi_2(\bar{p}) = v_2^{rs}(\bar{p})$. To show that such a $\bar{p}$ indeed exists, we argue that $\psi_2(\underline{p}) > 0 = v_2^{rs}(\underline{p})$, and $v_2^{rs}(1) \geq \psi_2(1)$ if and only if $\frac{A^2}{36B}\left[8\frac{1-\alpha}{\lambda_1+r} + 9\frac{2\alpha-1}{r}\right] \geq \frac{s}{\lambda_2 r}$. Hence, $\bar{p}$ is well defined, if and only if this inequality holds.

For a given value of s, define α_1 such that $\frac{A^2}{36B}\left[8\frac{1-\alpha_1}{\lambda_1+r} + 9\frac{2\alpha_1-1}{r}\right] = \frac{s}{\lambda_2 r}$, and define $\alpha^* = \max\{\frac{1}{2}, \alpha_1\}$. Since $\frac{A^2}{36B}\left[8\frac{1-\alpha}{\lambda_1+r} + 9\frac{2\alpha-1}{r}\right]$ is strictly increasing in α, for all $\alpha > \alpha^*$, we will have $\frac{A^2}{36B}\left[8\frac{1-\alpha}{\lambda_1+r} + 9\frac{2\alpha-1}{r}\right] > \frac{s}{\lambda_2}$. The assumption $\frac{s}{\lambda_2} < \frac{A^2}{4B}$ ensures that $\alpha_1 < 1$.

Next, define p_1' by $v_1^{rs}(p_1') = \psi_1(p_1')$ and $\check{p}_1$ by $\psi_1(\check{p}_1) = 0$, i.e., $\check{p}_1 = \frac{s}{\lambda_1 \frac{A^2(2\alpha-1)}{4B}}$. As argued before, given that $\bar{p}$ is well defined, an equilibrium in cutoff strategies exists if $p_1' < \bar{p}$.

For $\alpha = 1$, we have $\check{p}_1 = \underline{p}$. As $\alpha \to 1$, $\check{p}_1 \to \underline{p}$ from the right. ψ_1 is strictly decreasing, $v_1^{rs}(\underline{p}) = 0$ and v_1^{rs} is strictly increasing and convex. Thus, for $\alpha = 1$, $\check{p}_1 = \underline{p} = p_1'$, while $v_2^{rs} = 0$. Hence, $\bar{p}$

satisfies $\psi_2(\bar{p}) = 0$. This gives us $\bar{p} = \frac{s}{\lambda_2 \frac{A^2(2\alpha-1)}{4B}}$. This implies $\bar{p} > \underline{p} = p_1^{'}$ for $\alpha = 1$. Thus, by continuity, there exists a $\alpha_2 < 1$ such that, for all $\alpha > \alpha_2$, $p_1^{'} < \bar{p}$.

Define $\bar{\alpha} = \max\{\alpha^*, \alpha_2\}$. Thus for all $\alpha > \bar{\alpha}$, $\bar{p}$ is well defined and $p_1^{'} < \bar{p}$, implying our strategies do in fact constitute an equilibrium in cutoff strategies.

I Proof of Proposition 7

Method R is most attractive when $p = 1$; i.e., if, in some equilibrium, both firms use method R for some beliefs, they will do so at $p = 1$. Thus, suppose that $p = 1$ and that both firms choose R in some equilibrium. This means that firm 2' s payoff is equal to $\frac{A^2}{36B}\frac{[(10\alpha-1)\lambda_2+8(1-\alpha)\lambda_1]}{r+\lambda_2+\lambda_1} - \frac{s}{r+\lambda_1+\lambda_2} \equiv X$. If firm 2 unilaterally deviates, it gets $\frac{A^2}{9B}2(1-\alpha)\frac{\lambda_1}{r+\lambda_1} \equiv Y$. Direct computation shows that for $\alpha = 1$, $Y - X < 0$, and for $\alpha = \frac{1}{2}$, $Y - X > 0$ as $s > \frac{r}{r+\lambda_1}\lambda_2\frac{A^2}{9B}$. Since $Y - X$ is strictly decreasing in α, there exists an $\alpha^{'} \in (\frac{1}{2}, 1)$, such that for $\alpha > \alpha^{'}$, $Y - X < 0$. As $\lambda_1 > \lambda_2$, for $\alpha > \alpha^{'}$, we have $\frac{A^2}{36B}\frac{[(10\alpha-1)\lambda_1+8(1-\alpha)\lambda_2]}{r+\lambda_2+\lambda_1} - \frac{s}{r+\lambda_1+\lambda_2} > \frac{A^2}{9B}2(1-\alpha)\frac{\lambda_2}{r+\lambda_2}$ as well, so that playing R is firm 1's best response to firm 2's playing R. Thus, we can conclude that, for $\alpha > \alpha^{'}$, both firms choosing R constitute mutual best responses. Hence, for both firms to choose R for some range of beliefs, it is necessary that $\alpha > \alpha' > \frac{1}{2}$.

J Proof of Proposition 8

In the current proof we will write the payoffs as function of both α and p.

$$v_2^{rs}(\alpha, p) = \frac{\lambda_0 \alpha}{r+\lambda_0} + \frac{\lambda_1 p}{r+\lambda_0+\lambda_1}[1-\alpha-\frac{\alpha\lambda_0}{r+\lambda_0}] + C(1-p)[\frac{1-p}{p}]^{\frac{r+\lambda_0}{\lambda_1}}$$

$$\Rightarrow v_2^{rs'}(\alpha, p) = \frac{\lambda_1}{r+\lambda_0+\lambda_1}[1-\alpha-\frac{\alpha\lambda_0}{r+\lambda_0}] - C[\frac{1-p}{p}]^{\frac{r+\lambda_0}{\lambda_1}}\{1+\frac{r+\lambda_0}{\lambda_1 p}\}$$

Let $\frac{\lambda_1}{r+\lambda_0+\lambda_1}[1-\alpha-\frac{\alpha\lambda_0}{r+\lambda_0}] = D$ and $[\frac{1-p}{p}]^{\frac{r+\lambda_0}{\lambda_1}}\{1+\frac{r+\lambda_0}{\lambda_1 p}\} = \Gamma(p)$. D is strictly decreasing in α, and $\Gamma(p)$ is a strictly decreasing function of p and independent of α. We know from our previous analysis that at $p_1^* = \frac{\lambda_0}{\lambda_1}$, irrespective of the value of α, $v_2^{rs} = \frac{\lambda_0}{r+2\lambda_0}$ and $v_2^{rs'}(p_1^*) = 0$. This implies we have

$$v_2^{rs'}(\alpha, p_1^*) = D - C\Gamma(p_1^*) = 0 \Rightarrow C = \frac{D}{\Gamma(p_1^*)}$$

This implies for all $p > p_1^*$, we have

$$v_2^{rs'}(p) = D - \frac{D}{\Gamma(p_1^*)}\Gamma(p) = D(1 - \frac{\Gamma(p)}{\Gamma(p_1^*)})$$

Since $\Gamma(p)$ is strictly decreasing in p, $\frac{\Gamma(p)}{\Gamma(p_1^*)} < 1$ for all $p > p_1^*$. As D is strictly decreasing in α, we can infer that for all $p > p_1^*$, $v_2^{rs'}(\alpha_1, p) < v_2^{rs'}(\alpha_2, p)$ if $\alpha_1 > \alpha_2$. Since for all α, $v_2^{rs}(\alpha, p_1^*) = \frac{\lambda_0}{r+2\lambda_0}$ and $v_2^{rs'}(p_1^*) = 0$, we can infer that for any $p > p_1^*$, $v_2^{rs}(\alpha_1, p) < v_2^{rs}(\alpha_2, p)$ if $\alpha_1 > \alpha_2$.

We have

$$v_2^{rr}(\alpha, p) = \frac{\alpha\lambda_2 + (1-\alpha)\lambda_1}{r + \lambda_1 + \lambda_2} p + C(1-p)[\frac{1-p}{p}]^{\frac{r}{\lambda_1+\lambda_2}}$$

where C is an integration constant. Let $\frac{\alpha\lambda_2+(1-\alpha)\lambda_1}{r+\lambda_1+\lambda_2} = X(\alpha)$. X is decreasing in α as $\lambda_1 > \lambda_2$

For $p \geq p_1^*$, we have $v_2(\alpha, p)$ as

$$v_2(\alpha, p) = \begin{cases} v_2^{rr}(\alpha, p) & \text{if } p \in (\hat{p}_2(\alpha), 1], \\ v_2^{rs}(\alpha, p) & \text{if } p \in (\frac{\lambda_0}{\lambda_1}, \hat{p}_2(\alpha)] \end{cases}$$

We will prove that if $\alpha_1 > \alpha_2$, $v_2(\alpha_1, p) < v_2(\alpha_2, p)$ for all $p > p_1^*$.

Monotonicity of $\hat{p}_2$ with respect to α implies $\hat{p}_2(\alpha_1) < \hat{p}_2(\alpha_2)$.

Since for all $p > p_1^*$, $v_2^{rs}(\alpha_1, p) < v_2^{rs}(\alpha_2, p)$, for all $p \in (p_1^*, \hat{p}_2(\alpha_1)]$, $v_2(\alpha_1, p) = v_2^{rs}(\alpha_1, p) < v_2^{rs}(\alpha_2, p) = v_2(\alpha_2, p)$

Suppose $\tilde{v_2}^{rr}(\alpha_2)$ be such that $\tilde{v_2}^{rr}(\alpha_2, \hat{p}_2(\alpha_1)) = v_2^{rs}(\alpha_2, \hat{p}_2(\alpha_1))$ and $\bar{v_2}^{rr}(\alpha_2)$ be such that $\bar{v_2}^{rr}(\alpha_2, \hat{p}_2(\alpha_1)) = v_2^{rs}(\alpha_1, \hat{p}_2(\alpha_1))$. Since $v_2^{rs}(\alpha_2, \hat{p}_2(\alpha_1)) > v_2^{rs}(\alpha_1, \hat{p}_2(\alpha_1))$, for all $p > \hat{p}_2(\alpha_1)$ we shall have $\bar{v_2}^{rr} < \tilde{v_2}^{rr}$. Next, we have $v_2^{rr}(\alpha_1, \hat{p}_2(\alpha_1)) = v_2^{rs}(\alpha_1, \hat{p}_2(\alpha_1)) = \bar{v_2}^{rr}(\alpha_2, \hat{p}_2(\alpha_1))$. We have

$$v_2^{rr'}(\alpha, p) = X_\alpha - C_\alpha[\frac{1-p}{p}]^{\frac{r}{\lambda_1+\lambda_2}}[1 + \frac{r}{(\lambda_1+\lambda_2)p}]$$

Since X_α is decreasing in α, we have $X_{\alpha_1} < \bar{X}_{\alpha_2}$. Then $v_2^{rr}(\alpha_1, \hat{p}_2(\alpha_1)) = \bar{v_2}^{rr}(\alpha_2, \hat{p}_2(\alpha_1))$ implies $C_{\alpha_1} > \bar{C}_{\alpha_2}$. From the expression of $v_2^{rr'}$ we can now conclude that for all $p > \hat{p}_2(\alpha_1)$ we have $\bar{v_2}^{rr'}(\alpha_2) > v_2^{rr'}(\alpha_1)$. This implies $v_2^{rr}(\alpha_1, p) < \bar{v_2}^{rr}(\alpha_2, p)$ for all $p > \hat{p}_2(\alpha_1)$. Hence, for all $p > \hat{p}_2(\alpha_1)$, we have $v_2^{rr}(\alpha_1, p) < \tilde{v_2}^{rr}(\alpha_2, p)$.

One can also conclude from here that $v_2^{rr}(\alpha_1)$ can never intersect $v_2^{rr}(\alpha_2)$ from below. This is because if at a p, $v_2^{rr}(\alpha_1) = v_2^{rr}(\alpha_2)$, since X is decreasing in α, we must have $C_{\alpha_1} > C_{\alpha_2}$ and hence for all p we have $v_2^{rr'}(\alpha_1) < v_2^{rr'}(\alpha_2)$.

For $\alpha = \alpha_2$, in equilibrium, given firm 1 is using the innovtive method, firm 2 finds it optimal to use S for all $p \in (p_1^*, \hat{p}_2(\alpha_2))$. This means for all $p \in (\hat{p}_2(\alpha_1), \hat{p}_2(\alpha_2))$, if $\tilde{v_2}^{rr}(\alpha_2) = v_2^{rs}(\alpha_2)$, the slope of $\tilde{v_2}^{rr}(\alpha_2)$ is lower than that of $v_2^{rs}(\alpha_2)$. This implies we must have $\tilde{v_2}^{rr}(\alpha_2)$ lying below $v_2^{rs}(\alpha_2)$ for all $p \in (\hat{p}_2(\alpha_1), \hat{p}_2(\alpha_2))$. Hence for all $p \in (\hat{p}_2(\alpha_1), \hat{p}_2(\alpha_2))$, $v_2^{rr}(\alpha_1) < v_2^{rs}(\alpha_2)$. Since $v_2^{rr}(\alpha_1)$ cannot intersect $v_2^{rr}(\alpha_2)$ from below, we have $v_2^{rr}(\alpha_1, p) < v_2^{rr}(\alpha_2, p)$ for all $p > \hat{p}_2(\alpha_2)$. This concludes the proof.

K Proof of Proposition 9

We will first show that there does not exist an equilibrium in cutoff strategies such that firms use different thresholds and the more productive firm uses a more pessimistic threshold. Suppose to the contrary that there exist thresholds $\underline{p} < \bar{p}$ such that both firms use method S on $[0, \underline{p}]$, firm 1 (2) uses R (S) on $(\underline{p}, \bar{p}]$ and both firms use R on $(\bar{p}, 1]$, and these actions constitute mutual best responses. Since firm 1 optimally switches to method S from R at $\underline{p}$, we have $\underline{p} = \frac{\lambda_0}{\lambda_1}\left[\frac{10\alpha-1-\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha)}{\Pi(10\alpha-1)-\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha)}\right]$. In the conjectured equilibrium,

$\bar{p}$ is implicitly defined by $v_2^{rs}(\bar{p}) = \psi_2(\bar{p})$.

For $\alpha > \frac{1}{2}$ and $\Pi > 1$, $v_2^{rs'}(\underline{p}) > 0 = v_1^{rs'}(\underline{p})$ and v_1^{rs} is strictly convex on $[\underline{p}, 1)$. Direct computation via the relevant ODEs shows that v_1^{rs} can only intersect v_2^{rs} from the above (below) at a belief p is $p < (>) \frac{\lambda_0}{\lambda_1 \Pi}$.

We define $p_1^{'}$ to be the belief such that $v_1^{rs}(p_1^{'}) = \psi_1(p_1^{'})$. Consider $\lambda_1 \to \lambda_2 = \lambda > \lambda_0$. In this case, $\psi_i \to \psi = \frac{A^2}{36B}\frac{2\lambda_0(10\alpha-1)-\lambda\Pi[18\alpha-9]p}{r+2\lambda_0}$. Since $\psi(\frac{\lambda_0}{\lambda_1\Pi}) = v^{ss}$, we can infer that $\bar{p} < p_1^{'} < \frac{\lambda_0}{\lambda_1\Pi}$. This implies that the method R becomes dominant for firm 2 at a belief which is strictly lower than the belief at which R becomes dominant for firm 1. Hence, the conjectured equilibrium in cutoff strategies cannot exist.

We will now argue that there does not exist any equilibrium in cutoff strategies where either firm uses the same threshold or the less productive firm uses a more pessimistic threshold.

Suppose there exists an equilibrium in cutoff strategies where both players use the same threshold $\tilde{p}$. There are two possibilities:

1. $\tilde{p} > \underline{p}$: In this case, for all $p \leq \tilde{p}$, $v_1 = v_2 = v^{ss}$. However, from our previous analysis, we know that the more productive firm would have an incentive to deviate in some right-neighbourhood of $\underline{p}$. Hence, we cannot have $\tilde{p} > \underline{p}$.

2. $\tilde{p} \leq \underline{p}$: If this is indeed an equilibrium, then at $p = \tilde{p}$ we shall have $v_1 = v_2 = v^{ss}$ and,in some right neighborhood of $\tilde{p}$, we must have $\psi_2 \leq v^{ss}$. Otherwise, firm 2 is not playing a best response in that right neighborhood of $\tilde{p}$.

We first evaluate $\psi_2 - v^{ss}$ at $p = \frac{\lambda_0}{\lambda_1\Pi}$. Direct computation shows that, at $p = \frac{\lambda_0}{\lambda_1\Pi}$, we have

$$\psi_2 - v^{ss} = \frac{\lambda_0 A^2}{36B}\left\{\frac{10\alpha\lambda_1 - 8\alpha\lambda_2 + 8\lambda_2 - \lambda_1}{r\lambda_2 + \lambda_0(\lambda_1+\lambda_2)} - \frac{7+2\alpha}{r+2\lambda_0}\right\}$$

$$= (\lambda_1 - \lambda_2)[r(10\alpha-1) + 9\lambda_0(2\alpha-1)] > 0$$

Since ψ_2 is strictly decreasing in p, and $\tilde{p} \leq \underline{p} < \frac{\lambda_0}{\lambda_1 \Pi}$, in some right neighborhood of $\tilde{p}$, we have $\psi_2 > v^{ss}$. Hence, we cannot have a symmetric equilibrium with a cutoff $\tilde{p} \leq \underline{p}$ if firms are asymmetric. Hence, we can conclude that a symmetric equilibrium in cutoff strategies does not exist.

Next, we will show that no equilibrium in cutoff strategies exists such that the less productive player uses a more pessimistic cutoff. Suppose there exists such an equilibrium, i.e., firm 2 leaves method R at a more pessimistic belief. This means that firm 2 is the last firm to use R, and hence it optimally switches to S at a belief $\tilde{p} = \frac{\lambda_0}{\lambda_2}[\frac{(10\alpha-1)-\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha)}{(10\alpha-1)\Pi-\frac{\lambda_0}{r+2\lambda_0}(7+2\alpha)}]$. Since $\tilde{p} > \underline{p}$ in this conjectured equilibrium, both firms will use the method S in some right-neighbourhood of $\underline{p}$, where both firms' value will equal v^{ss}—implying that firm 1 has an incentive to deviate in some right-neighbourhood of $\underline{p}$. Hence, the conjectured equilibrium does not exist.

L Proof of Proposition 10

We will first show that for extremely asymmetric firms, for high values of r, ψ_1 is strictly below v^{ss} for all p. To see this, first consider $\lambda_0 < \lambda_1$, and $\lambda_1 \to \infty$ for a given value of λ_2, λ_0 and other parameters. From the expression of ψ_1, we see that $\psi_1 \to \frac{A^2}{36B}\frac{\lambda_0(10\alpha-1)-\lambda_2\Pi[18\alpha-9]p}{r+\lambda_0}$.

Next, suppose $\lambda_0 > \lambda_1$. For given values of λ_0 and λ_1, consider values of Π such that, as $\lambda_2 \to 0$, $\lambda_2\Pi$ remain constant. Direct computation shows that the limiting value of ψ_1 in this case is same as in the preceding paragraph.

At $p=0$, the limiting value of $\psi_1 = \frac{A^2}{36B}\frac{\lambda_0(10\alpha-1)}{r+\lambda_0} \equiv \underline{\psi_1}$. Direct computation shows

$$v^{ss} - \underline{\psi_1} = \frac{\lambda_0 A^2}{36B}[\frac{7+2\alpha}{r+2\lambda_0} - \frac{(10\alpha-1)}{r+\lambda_0}] > 0$$

for all $r \geq \frac{9\lambda_0(2\alpha-1)}{8(1-\alpha)} \equiv \bar{r}(\alpha)$. Since ψ_1 is strictly negatively sloped, we can conclude that, as $\lambda_1 \to \infty$, for all p, ψ_1 is strictly below v^{ss}.

Next, we will show that, for extremely asymmetric firms, for any value of r, ψ_2 is strictly higher than v^{ss} at $p = \underline{p}$. From the expression of ψ_2, we can infer that its limiting value as $\lambda_1 \to \infty$ is the same as the limiting value for$\lambda_2 \to 0$ such that $\lambda_2\pi$ remain constant. This common limiting value is equal to $\frac{A^2}{36B}\frac{\lambda_0(10\alpha-1)-\lambda_2\Pi[18\alpha-9]p}{\lambda_0}$, which at $p=0$ is equal to $\frac{A^2}{36B}(10\alpha-1) \equiv \underline{\psi_2}$. Direct computation shows

$$\underline{\psi_2} - v^{ss} = \frac{A^2}{36B}[\frac{2\alpha[5r+9\lambda_0]-[r+9\lambda_0]}{(r+2\lambda_0)} > \frac{A^2}{36B}[\frac{2\alpha[r+9\lambda_0]-[r+9\lambda_0]}{(r+2\lambda_0)} = \frac{A^2}{36B}[\frac{[r+9\lambda_0](2\alpha-1)}{(r+2\lambda_0)} \geq 0$$

Hence, as $\lambda_1 \to \infty$, or $\lambda_2 \to 0$ s.t. $\lambda_2\Pi$ remaining constant, $\psi_2(\underline{p}) > v^{ss}$. This ensures that $\bar{p}$ is bounded above $\underline{p}$ for highly hetereogeneous firms.

In any conjectured equilibrium in cutoff strategies, v_1 is greater than or equal to v^{ss} for all p. Hence, we can conclude that, for $r \geq \bar{r}(\alpha)$,

1. If $\lambda_1 > \lambda_0$, we can find a $\bar{\lambda_1} > \lambda_2$ such that, for all $\lambda_1 > \bar{\lambda_1}$, $p_1^{'} < \bar{p}$ implying the existence of an equilibrium in cutoff strategies.

2. If $\lambda_1 < \lambda_0$, we can find a $\bar{\lambda_2} \in (0, \lambda_1)$ such that for all $\lambda_2 < \bar{\lambda_2}$, $p_1^{'} < \bar{p}$ implying the existence of an equilibrium in cutoff strategies.